# NELSON SENIOR MATHS METHODS 12
## FULLY WORKED SOLUTIONS

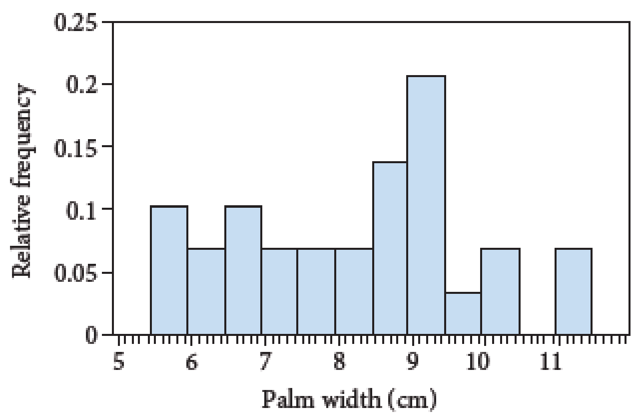
## Chapter 8 Continuous random samples and the normal distribution
### Exercise 8.01 Continuous random variables and probability distributions

Concepts and techniques

**1** **a**

| Width (cm) | Frequency | Relative frequency. |
|---|---|---|
| 5.5–5.9 | 3 | 0.103 |
| 6–6.4 | 2 | 0.069 |
| 6.5–6.9 | 3 | 0.103 |
| 7–7.4 | 2 | 0.069 |
| 7.5–7.9 | 2 | 0.069 |
| 8–8.4 | 2 | 0.069 |
| 8.5–8.9 | 4 | 0.138 |
| 9–9.4 | 6 | 0.207 |
| 9.5–9.9 | 1 | 0.034 |
| 10–10.4 | 2 | 0.069 |
| 10.5–10.9 | 0 | 0 |
| 11–11.4 | 2 | 0.069 |
| Total | 29 | 0.999 |

**b**





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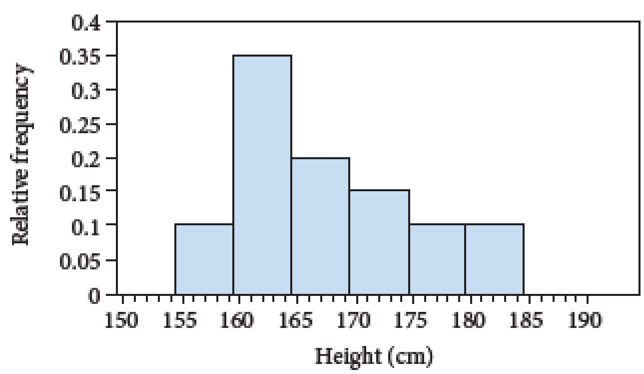
**c** $P(6 \le w < 7) = 0.069 \times 0.9 + 0.103 \times 1 + 0.069 \times 0.1 = 0.172$

**d** $P(7.5 \le w < 8.5) = 0.069 \times 0.9 + 0.069 \times 1 + 0.1 \times 0.138 \approx 0.145$

**2** **a**

| Height (cm) | Frequency | Relative frequency |
|---|---|---|
| 155–159 | 2 | 0.1 |
| 160–164 | 7 | 0.35 |
| 165–169 | 4 | 0.2 |
| 170–174 | 3 | 0.15 |
| 175–179 | 2 | 0.1 |
| 180–184 | 2 | 0.1 |

**b**




**c** 161–164 is actually 160.5 to 164.5

$P(160.5 \le h \le 164.5) = \frac{4}{5} \times 0.35 = 0.28$

**d** Over 168 is actually over 168.5

$P(h > 168.5) = \frac{1}{5} \times 0.20 + 0.15 + 0.1 + 0.1 = 0.39$

**e** Strictly between 165 and 175 is actually 165.5–174.5

$P(165.5 < h < 174.5) = \frac{4}{5} \times 0.20 + 0.15 = 0.31$

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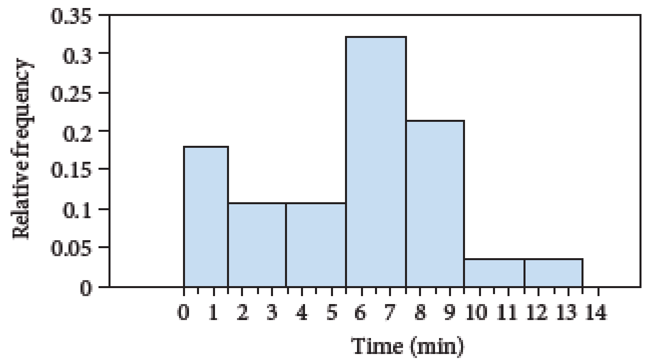
**3** **a**

| Time (min) | Frequency | Relative frequency |
|---|---|---|
| 0–1 | 5 | 0.178 571 4 |
| 2–3 | 3 | 0.107 142 9 |
| 4–5 | 3 | 0.107 142 9 |
| 6–7 | 9 | 0.321 428 6 |
| 8–9 | 6 | 0.214 285 7 |
| 10–11 | 1 | 0.035 714 3 |
| 12–13 | 1 | 0.035 714 3 |

**b**




**c** $P(T < 4.5) = 0.178\ 571\ 4 + 0.107\ 142\ 9 + \frac{1}{2} \times 0.107\ 142\ 9 = 0.339\ 285\ 8$

**d** $P(T > 10.5) = \frac{1}{2} \times 0.035\ 714\ 3 + 0.035\ 714\ 3 = 0.053\ 571\ 45$

**e** $P(5 < t < 10) = P(5.5 < h < 9.5) = 0.321\ 428\ 6 + 0.214\ 285\ 7 = 0.535\ 714\ 3$

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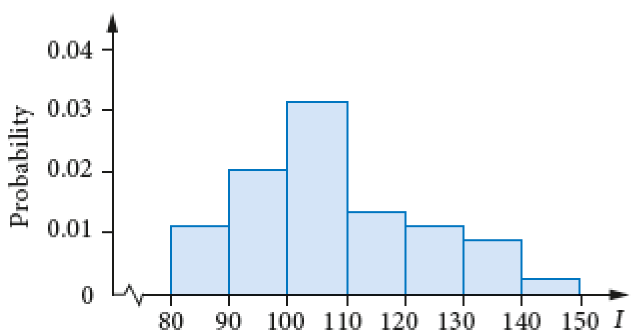
## Reasoning and communication

**4** **a**

| IQ | Frequency | Relative frequency |
|---|---|---|
| 80–89 | 5 | 0.113 |
| 90–99 | 9 | 0.205 |
| 100–109 | 14 | 0.318 |
| 110–119 | 6 | 0.136 |
| 120–129 | 5 | 0.114 |
| 130–139 | 4 | 0.091 |
| 140–149 | 1 | 0.023 |
| **Total** | 44 | 1 |




**b** $P(99.5 \leq IQ < 110.5) = 0.318 + 0.1 \times 0.136 = 0.3316$

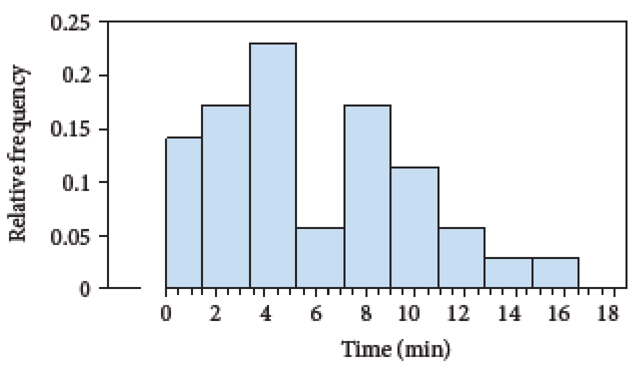
**c** $P(100.5 \leq IQ < 109.5) = P(100 \leq IQ < 109) = \dfrac{9}{10} \times 0.318 = 0.2862$

**d** 107.45

**e** This group is a sample and will not necessarily have the same mean as the population.

**5** **a**

| Time (min) | Frequency | Relative frequency |
|---|---|---|
| 0–1 | 5 | 0.142 857 1 |
| 2–3 | 6 | 0.171 428 6 |
| 4–5 | 8 | 0.228 571 4 |
| 6–7 | 2 | 0.057 142 9 |
| 8–9 | 6 | 0.171 428 6 |
| 10–11 | 4 | 0.114 285 7 |
| 12–13 | 2 | 0.057 142 9 |
| 14–15 | 1 | 0.028 571 4 |
| 16–17 | 1 | 0.028 571 4 |



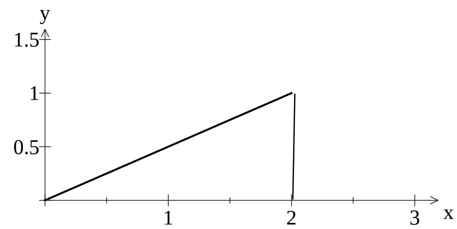

**b** $P(t < 5) = 0.142\ 857\ 1 + 0.171\ 428\ 6 + \frac{1}{2} \times 0.228\ 571\ 4 \approx 0.429$

**c** They come at intervals of less than 16 minutes.

## Exercise 8.02 Probability density and cumulative distribution functions

### Concepts and techniques

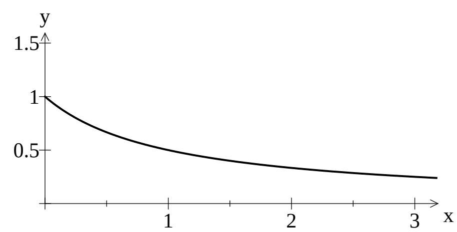
**1** **a** $f(x) = 0.5x$ for the interval [0, 2].




Area = $0.5 \times (2 \times 1) = 1$

Yes, could be a probability density function.

**b** $f(x) = \dfrac{1}{(x+1)^2}$ for the interval $[0, \infty)$.




$$\text{Area} = \int_0^\infty \frac{1}{(x+1)^2}\,dx = -\left[(x+1)^{-1}\right]_0^\infty = -\left[\frac{1}{(x+1)}\right]_0^\infty = -(0-1) = 1$$

Yes, could be a probability density function.

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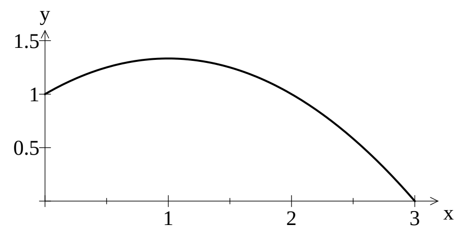
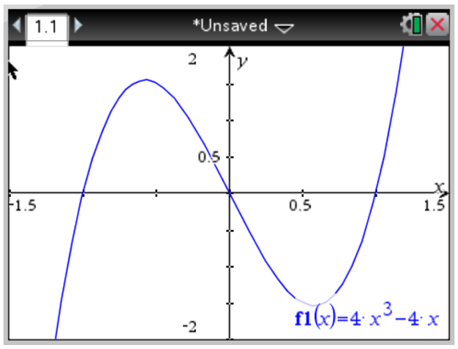
**c** $f(x) = \frac{1}{3}(3 - x)(x + 1)$ for the interval [0, 3].




$$\text{Area} = \frac{1}{3}\int_0^3 (3 - x)(x + 1)\,dx = 3$$

No, could not be a probability density function as area ≠ 1.

**d** TI-Nspire CAS

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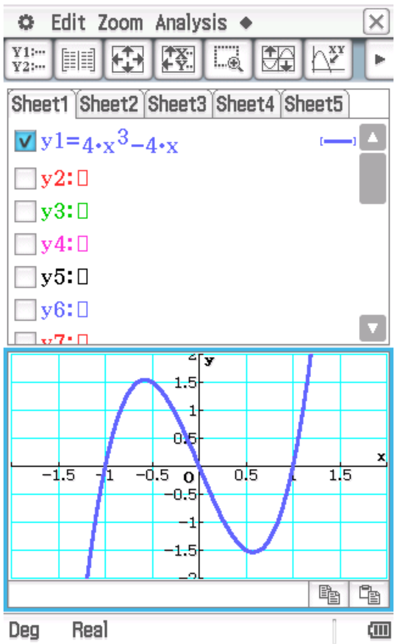



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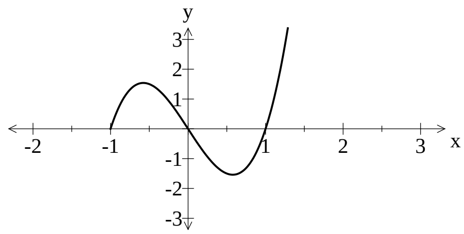
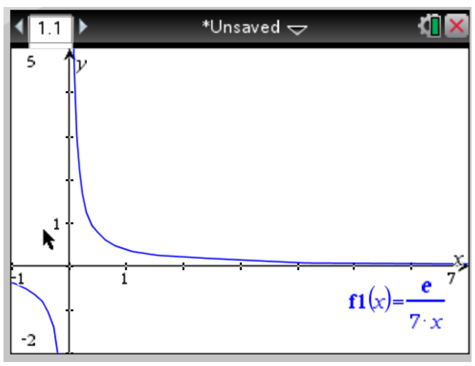
$f(x) = 4x^3 - 4x$ for the interval $[-1, \sqrt{2}]$



The function has negative values in the given domain. $P(x) \geq 0$.

No, could not be a probability density function.

**e** TI-Nspire CAS





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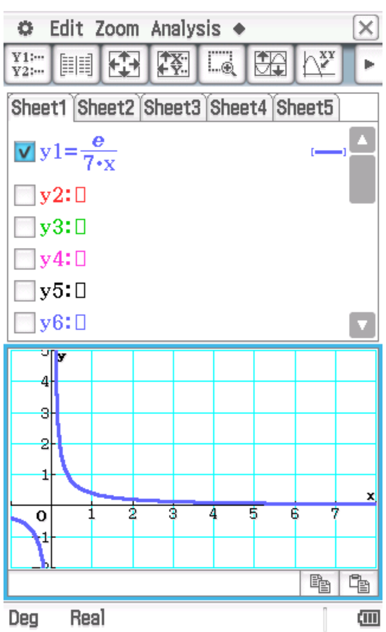
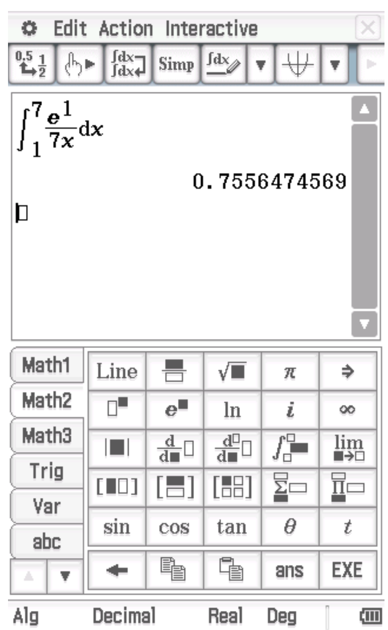
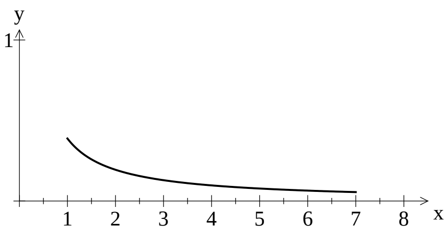


$f(x) = \dfrac{e}{7x}$ for the interval [1, 7]




$$\text{Area} = \frac{e}{7}\int_1^7 \frac{1}{x}\,dx = \frac{e}{7}\big[\ln(x)\big]_1^7 = \frac{e}{7}\big(\ln(7) - \ln(1)\big) = 0.755...$$

No, could not be a probability density function as area ≠ 1.

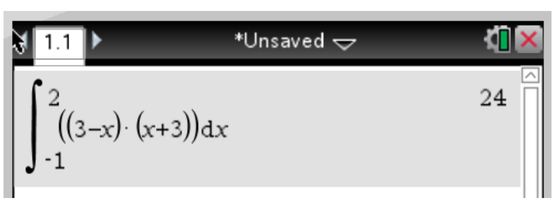
**2** **a** $\displaystyle\int_5^\infty \frac{1}{(x-1)^2}\,dx = -\Big[(x-1)^{-1}\Big]_5^\infty = -\left(0 - \frac{1}{4}\right) = \frac{1}{4}$

$f(x) = \dfrac{4}{(x-1)^2}$ is a pdf on [0, ∞).

**b** $\displaystyle\int_0^4 x^3\,dx = \left[\frac{x^4}{4}\right]_0^4 = \frac{1}{4}(256 - 0) = 64$

$f(x) = \dfrac{x^3}{}$ is a pdf on [0, 4].

**c** **TI-Nspire CAS**

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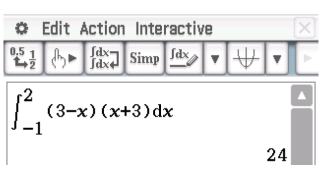


$$\int_{-1}^{2}(3-x)(x+3)\,dx = 24$$

$f(x) = \dfrac{(3-x)(x+3)}{24}$ is a pdf on [–1, 2].

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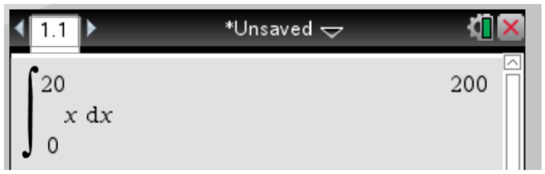
**d** TI-Nspire CAS

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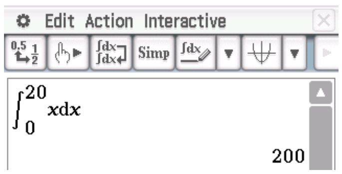
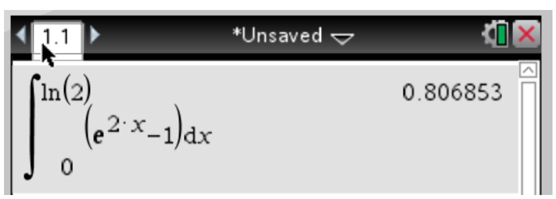



$$\int_0^{20} x\,dx = 200$$

$f(x) = \dfrac{x}{200}$ is a pdf on [0, 20].

**e** TI-Nspire CAS

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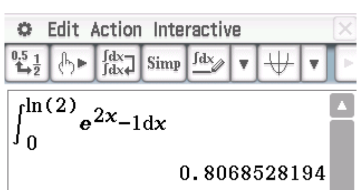


$$\int_0^{\ln(2)} \left(e^{2x}-1\right)dx = \left[\frac{e^{2x}}{2} - x\right]_0^{\ln(2)} = \left(\frac{e^{2\ln(2)}}{2} - \ln(2)\right) - \left(\frac{1}{2}\right) = 2 - \ln(2) - \frac{1}{2} = \frac{3 - 2\ln(2)}{2}$$

where $e^{\ln(4)} = 4$

$f(x) = \dfrac{2\left(e^{2x}-1\right)}{3-2\ln(2)}$ is a pdf on [0, ln (2)].


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**3** **a** $$\int_1^x x^{-2}dx = -\left[\frac{1}{x}\right]_1^x = -\left(\frac{1}{x} - 1\right) = 1 - \frac{1}{x}$$

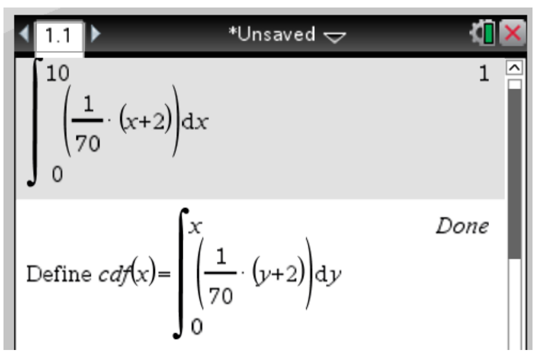
**b** $$P(1 < X < 2) = \left(1 - \frac{1}{2}\right) - (1 - 1) = \frac{1}{2}$$

**c** $$P(2 < X < 3) = \left(1 - \frac{1}{3}\right) - \left(1 - \frac{1}{2}\right) = \frac{1}{6}$$

**d** $$P(2 < X < 4) = \left(1 - \frac{1}{4}\right) - \left(1 - \frac{1}{2}\right) = \frac{1}{4}$$

**e** $$P(3 < X < 4) = P(2 < X < 4) - P(2 < X < 3) = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

**4** TI-Nspire CAS



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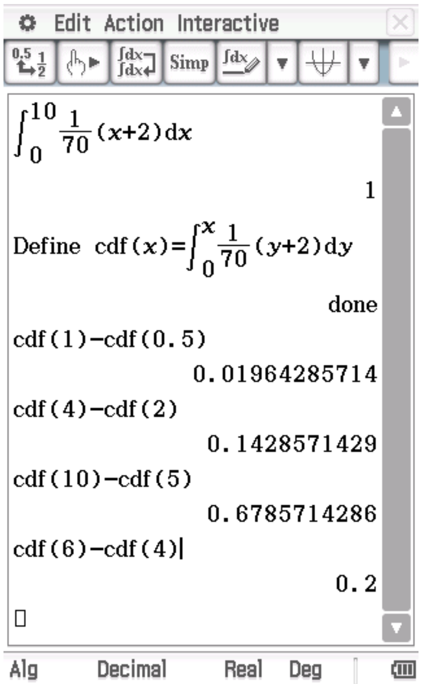


$f(x) = \frac{1}{70}(x + 2)$ defined on the interval [0, 10].

**a** $\int_0^x \frac{1}{70}(x+2)dx = \frac{1}{70}\left[\frac{x^2}{2} + 2x\right]_0^x = \frac{x^2+4x}{140}$

**b** $P(0.5 < X < 1) = \left[\frac{x^2+4x}{140}\right]_{0.5}^{1} = \frac{1}{140}(5-2.25) = \frac{2.75}{140} = \frac{11}{560}$

**c** $P(2 < X < 4) = \left[\frac{x^2+4x}{140}\right]_{2}^{4} = \frac{1}{140}(32-12) = \frac{20}{140} = \frac{1}{7}$

**d** $P(5 < X < 10) = \left[\frac{x^2+4x}{140}\right]_{5}^{10} = \frac{1}{140}(140-45) = \frac{95}{140} = \frac{19}{28}$

**e** $P(4 < X < 6) = \left[\frac{x^2+4x}{140}\right]_{4}^{6} = \frac{1}{140}(60-32) = \frac{28}{140} = \frac{1}{5}$

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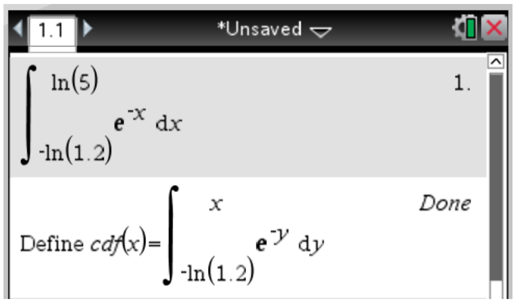
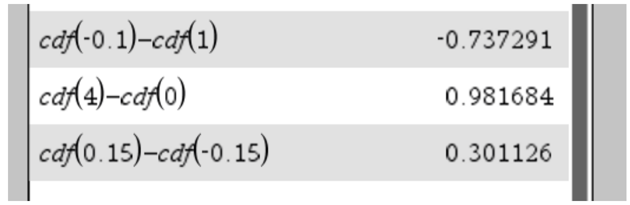
**5** **TI-Nspire CAS**

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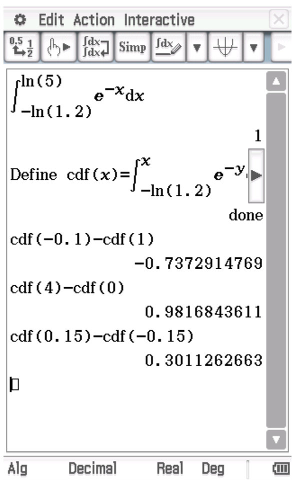


$f(x) = e^{-x}$ defined on the interval $[-\ln(1.2), \ln(5)] \approx [-0.1823, 1.6094]$.

**a** $\int_{-\ln(1.2)}^{x} e^{-x}dx = \left[-e^{-x}\right]_{-\ln(1.2)}^{x} = -e^{-x} - (-e^{-(-\ln(1.2))}) = 1.2 - e^{-x}$

**b** $P(2 < X < 3) = 0$, outside domain

**c** $P(-0.1 < X < 1) = 1.2 - e^{-1} - (1.2 - e^{-0.1}) \approx 0.737$

**d** $P(0 < X < 4) = -\left[e^{-x}\right]_{0}^{\ln(5)} = -e^{-\ln(5)} + 1 = -0.2 + 1 = 0.8$

**e** $P(-0.15 < X < 0.15) = -\left[e^{-x}\right]_{-0.15}^{0.15} = -\dfrac{1}{e^{0.15}} + e^{0.15} \approx 0.301$

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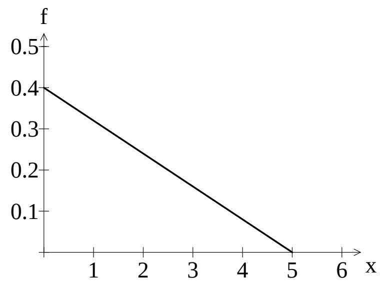
## Reasoning and communication

**6** $F(x) = 0.4x - 0.04x^2$ for [0, 5].

$f(x) = F'(x) = 0.4 - 0.08x$



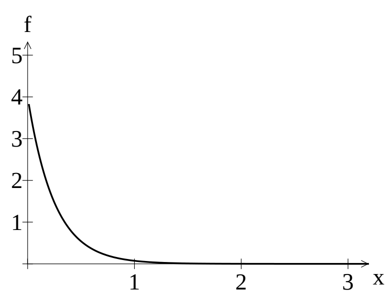

$f(x) > 0$ for [0, 5]

Area of triangle = 0.5 × 0.4 × 5 = 1

∴ pdf

**7** $F(t) = 1 - e^{-4t}$ for $[0, \infty)$.

$f(x) = F'(x) = 4e^{-4t}$



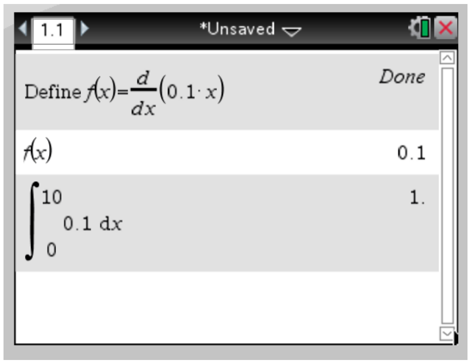

$f(x) > 0$ for $x > 0$

Area under the curve for $x > 0$

$$\left[1 - e^{-4t}\right]_0^\infty = \left(1 - \frac{1}{e^\infty}\right) - (1 - 1) = 1$$

∴ pdf

**8** TI-Nspire CAS

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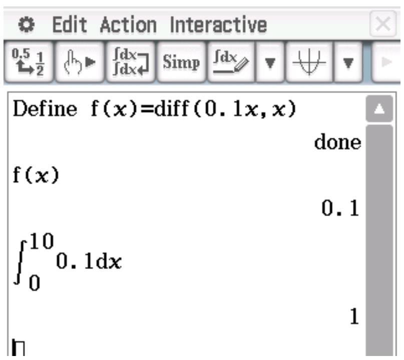
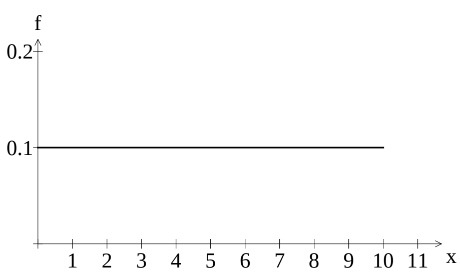



$F(t) = 0.1x$ for [0, 10].

$f(x) = F'(x) = 0.1$




$f(x) > 0$ for $x > 0$

Area under the curve = $10 \times 0.1 = 1$

$\therefore$ pdf

## Exercise 8.03 Simple continuous random variables

### Concepts and techniques

**1** **a** [0, 20]

$20 \times p(x) = 1$

$p(x) = \frac{1}{20}$

**b** [0, 18]

$18 \times p(x) = 1$

$p(x) = \frac{1}{18}$

**c** [10, 20]

$(20 - 10) \times p(x) = 1$

$p(x) = \frac{1}{10}$

**d** [5, 15]

$(15 - 5) \times p(x) = 1$

$p(x) = \frac{1}{10}$

**e** [6, 36]

$30 \times p(x) = 1$

$p(x) = \frac{1}{30}$

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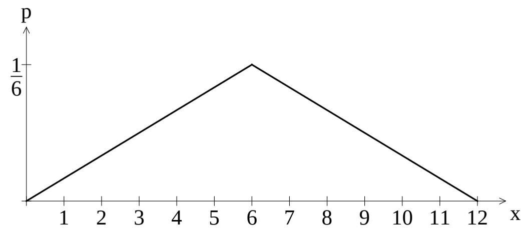
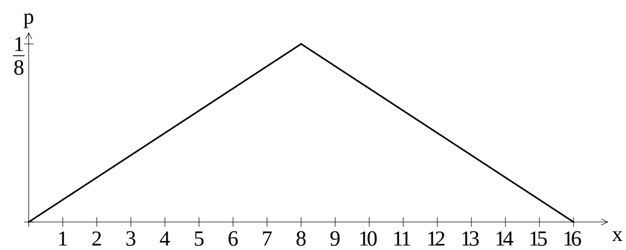
**2** **a** [0, 12], centre height = $\frac{1}{6}$, $m$ (0–6) = $\frac{1}{6^2}$, $m$ (6–12) = $-\frac{1}{6^2}$



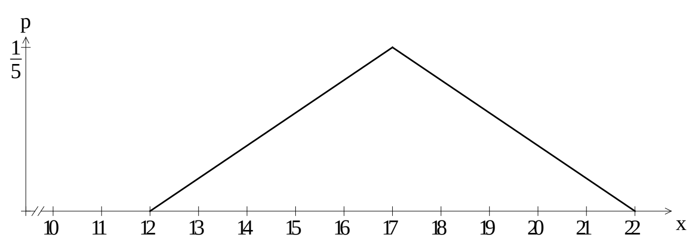

$$p(x) = \begin{cases} \frac{x-0}{36} = \frac{x}{36} & \text{for } 0 \le x \le 6 \\ -\frac{(x-12)}{36} = -\frac{x}{36} + \frac{1}{3} & \text{for } 6 \le x \le 12 \end{cases}$$

**b** [0, 16]




$$p(x) = \begin{cases} \frac{x-0}{64} & \text{for } 0 \le x \le 8 \\ -\frac{x-16}{64} = -\frac{x}{64} + \frac{1}{4} & \text{for } 8 \le x \le 16 \end{cases}$$

**c** [12, 22], width of base is 10




$$p(x) = \begin{cases} \frac{(x-12)}{25} & \text{for } 12 \le x \le 17 \\ -\frac{(x-22)}{25} & \text{for } 17 \le x \le 22 \end{cases}$$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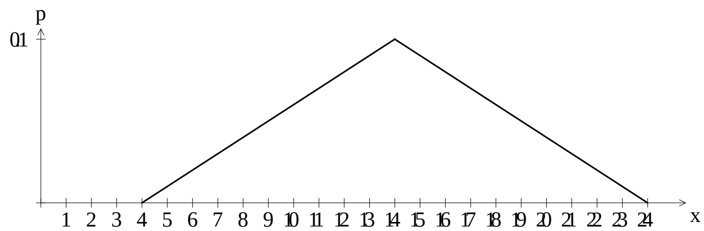
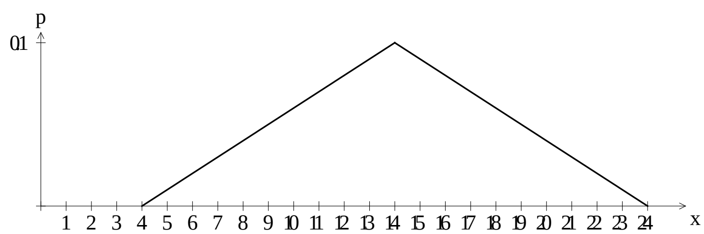
**d** [4, 24], width of base is 20



$$p(x)=\begin{cases}\dfrac{(x-4)}{100} & \text{for} \quad 4\le x\le 14\\ -\dfrac{(x-24)}{100} & \text{for} \quad 14\le x\le 24\end{cases}$$

**e** [2, 34], width of base is 20



$$p(x)=\begin{cases}\dfrac{(x-2)}{256} & \text{for} \quad 2\le x\le 18\\ -\dfrac{(x-34)}{256} & \text{for} \quad 18\le x\le 34\end{cases}$$

**3** **a** The maximum value of $p(x)$ is $\dfrac{1}{3}$

**b** The slope of the line on the left of 6 is $\dfrac{\text{rise}}{\text{run}}=\dfrac{\frac{1}{3}}{2}=\dfrac{1}{6}$

**c** The slope of the line on the right of 6 is $\dfrac{\text{rise}}{\text{run}}=\dfrac{-\frac{1}{3}}{4}=-\dfrac{1}{12}$

**d** $$p(x)=\begin{cases}\dfrac{x-4}{6} & \text{for} \quad 4\le x\le 6\\ -\dfrac{x-10}{12} & \text{for} \quad 6\le x\le 10\end{cases}$$

**4** **a** [5, 15] with maximum value at 7.

$$p(x)=\begin{cases}\dfrac{x-5}{10} & \text{for } 5\le x\le 7\\ -\dfrac{x-15}{40} & \text{for } 7\le x\le 15\end{cases}$$

**b** [4, 10] with maximum value at 8.

$$p(x)=\begin{cases}\dfrac{x-4}{12} & \text{for } 4\le x\le 8\\ -\dfrac{x-10}{6} & \text{for } 8\le x\le 10\end{cases}$$

**c** [20, 30] with maximum value at 23.

$$p(x)=\begin{cases}\dfrac{x-20}{15} & \text{for } 20\le x\le 23\\ -\dfrac{x-30}{35} & \text{for } 23\le x\le 30\end{cases}$$

**d** [0, 20] with maximum value at 15.

$$p(x)=\begin{cases}\dfrac{x}{150} & \text{for } 0\le x\le 15\\ -\dfrac{x-20}{50} & \text{for } 15\le x\le 20\end{cases}$$

**e** [20, 90] with maximum value at 60.

$$p(x)=\begin{cases}\dfrac{x-20}{1400} & \text{for } 20\le x\le 60\\ -\dfrac{x-90}{1050} & \text{for } 60\le x\le 90\end{cases}$$

## Reasoning and communication

**5** **a** $p(t)=\dfrac{1}{5}$ for $15\le x\le 20$

**b** $P(x > 16) = 0.8$

**c** $P(x > 18) = 0.4$

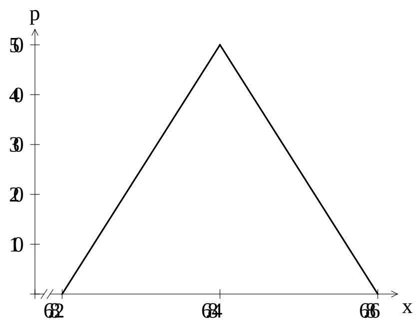
**6** **a** 0 and 10 minutes

**b** $P$(between 0 and 5 minutes after getting to the stop) = 0.5

**c** $b(t) = 0.1$

**d** $P$(Carol getting a bus within 3 minutes of arriving at the stop) = 0.3

**7** [6.82, 6.86], width of base is 0.04.




$$p(x) = \begin{cases} 2500x - 17\ 050 & \text{for} \quad 6.82 \le x \le 6.84 \\ -2500x + 17\ 150 & \text{for} \quad 6.84 \le x \le 6.86 \end{cases}$$

$P(x < 6.85 \text{ m}) = \int_{6.84}^{6.85} -2500x + 17\ 150\ dx + \int_{6.82}^{6.84} 2500x - 17\ 050\ dx = 0.375 + 0.5 = 0.875$

**8** 102% of 82 = 83.64

98% of 82 = 80.36

Width of base is 3.28

$$p(x) = \begin{cases} \dfrac{x - 80.36}{1.64^2} & \text{for} \quad 80.36 \le x \le 82 \\ -\dfrac{x - 83.64}{1.64^2} & \text{for} \quad 82 \le x \le 83.64 \end{cases}$$

$P(81 < x < 83 \text{ m}) = \int_{81}^{82} \frac{x - 80.36}{1.64^2} dx + \int_{82}^{83} -\frac{x - 83.64}{1.64^2} dx$

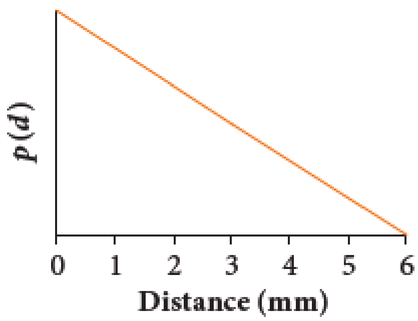
$= 2\left[\frac{x^2 - 160.72x}{2 \times 1.64^2}\right]_{81}^{82}$ by symmetry

$= 2 \times 0.4238\ldots$

$= 0.0877\ldots$

The probability that someone whose bathroom scales show them as weighing 82 kg (between 81 and 83 kg) is about 0.848.

**9** **a**




**b** $0.5 \times h(0) \times 6 = 1$, $h(0) = \frac{1}{3}$

$m = -\frac{\frac{1}{3}}{6} = -\frac{1}{18}$

so $p(d) = -\frac{1}{18}(d - 6) = \frac{1}{18}(6 - d)$

**c** Since the player will be within the horizontal part of the triple twenty, it is only the vertical distance that matters.

$P(d \le 4) = \int_0^4 \frac{1}{18}(6 - x)dx = \frac{1}{18}\left[6x - \frac{x^2}{2}\right]_0^4 = \frac{8}{9}$

**d** It will reduce the target area by the area of the dart shaft, but by shifting his target to a point 4 mm to the (larger) side of the existing dart, it will make no difference to the probability of getting a triple 20 on the second dart.

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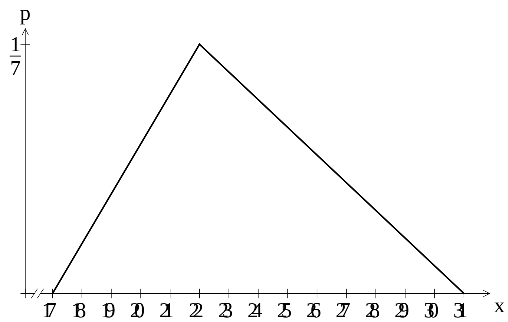
**10** **a** [17, 31], width of base is 14.




$$p(x)=\begin{cases}\dfrac{x}{35}-\dfrac{17}{35} & \text{for } 17\le x\le 22\\ -\dfrac{x}{63}+\dfrac{31}{63} & \text{for } 22\le x\le 31\end{cases}$$

**b** $P(18 < x < 20) = \int_{18}^{20}\frac{x}{35}-\frac{17}{35}\,dx = 0.114$

**c** $P(24.5 < x < 25.5) = \int_{24.5}^{25.5}-\frac{x}{63}+\frac{31}{63}\,dx = 0.0952$

**d** $P(x < 25) = \int_{22}^{25}-\frac{x}{63}+\frac{31}{63}\,dx+\int_{17}^{22}\frac{x}{35}-\frac{17}{35}\,dx=\frac{5}{14}+\frac{5}{14}=\frac{5}{7}$

**e** Depends how often she can afford to be late. 80% of time? 10% of time?

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## Exercise 8.04 Expected value

### Concepts and techniques

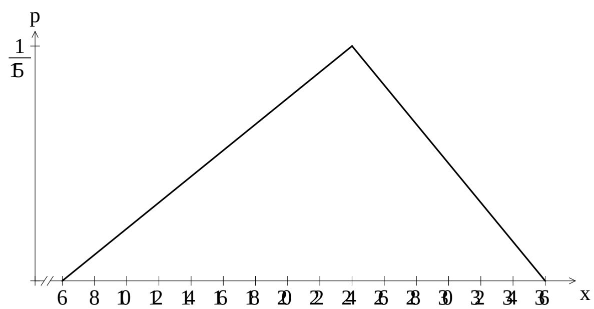
**1** **a** $f(x)=\frac{1}{24}$

**b** $E(x)=\int_3^{27} x\times\frac{1}{24}dx=\frac{1}{48}\left[x^2\right]_3^{27}=\frac{1}{48}(729-9)=15$

**2** $E(x)=\int_{20}^{90}\frac{x}{70}dx=\frac{1}{140}\left[x^2\right]_{20}^{90}=\frac{1}{140}(8100-400)=55$

**3** $E(x)=\int_8^{36}\frac{x}{28}dx=\frac{1}{56}\left[x^2\right]_8^{36}=\frac{1}{56}(1296-64)=22$

**4** [6, 36], width of base is 30.




$$p(x)=\begin{cases}\frac{x}{270}-\frac{1}{45} & \text{for } 6\le x\le 24\\ -\frac{x}{180}+\frac{1}{5} & \text{for } 24\le x\le 36\end{cases}$$

$$E(x)=\int_6^{24}x\times\left(\frac{x}{270}-\frac{1}{45}\right)dx+\int_{24}^{36}x\times\left(-\frac{x}{180}+\frac{1}{5}\right)dx$$

$$=\left[\frac{x^3}{810}-\frac{x^2}{90}\right]_6^{24}+\left[-\frac{x^3}{540}+\frac{x^2}{10}\right]_{24}^{36}$$

$$=10.8+11.2$$

$$=20$$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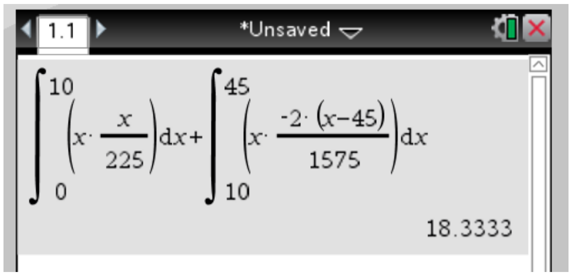
**5** TI-Nspire CAS

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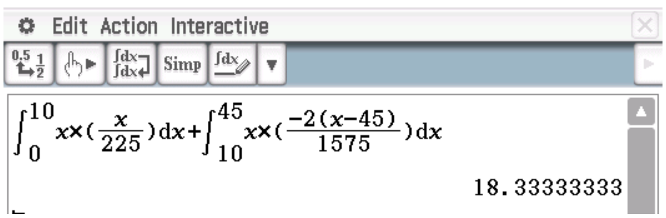
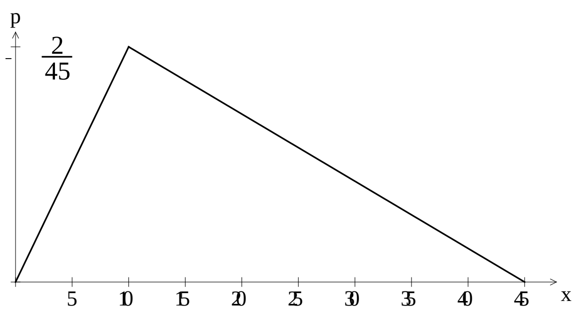


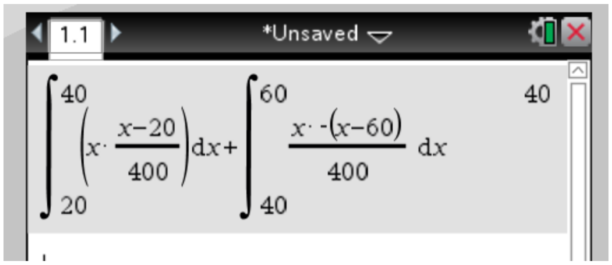

[0, 45], width of base is 45




$$p(x)=\begin{cases}\dfrac{x}{225} & \text{for } 0\le x\le 10\\[2ex] \dfrac{-2(x-45)}{1575} & \text{for } 10\le x\le 45\end{cases}$$

$$E(x)=\int_0^{10} x\times\left(\frac{x}{225}\right)dx+\int_{10}^{45} x\times\left(\frac{-2(x-45)}{1575}\right)dx = 18\tfrac{1}{3}$$

**6** TI-Nspire CAS

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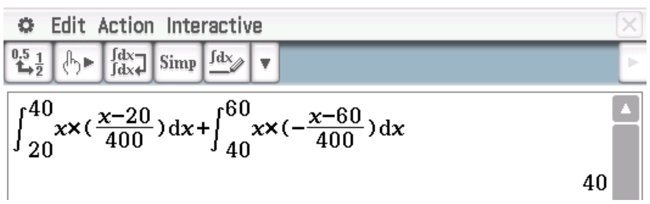
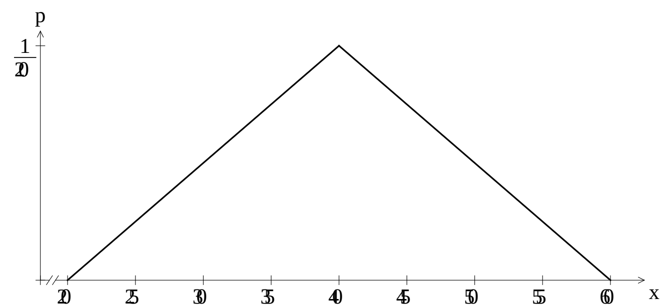



[20, 60], width of base is 40




$$p(x)=\begin{cases}\dfrac{x-20}{400} & \text{for} \quad 20\le x\le 40\\[2ex] \dfrac{-(x-60)}{400} & \text{for} \quad 40\le x\le 60\end{cases}$$

$$E(x)=\int_{20}^{40} x\times\left(\frac{x-20}{400}\right)dx+\int_{40}^{60} x\times\left(-\frac{x-60}{400}\right)dx$$

$$=40$$

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## Reasoning and communication

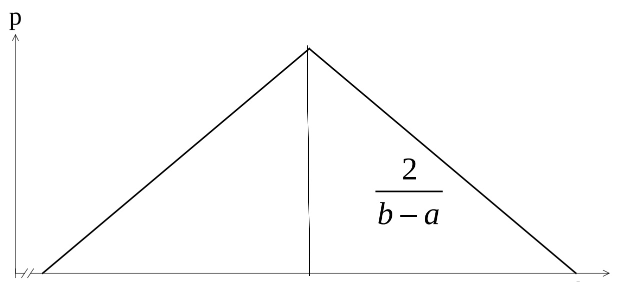
**7** [$a$, $b$], width of base is $b - a$

$$a+\frac{b-a}{2}=\frac{a+b}{2}$$

Area of triangle = 1

$$\frac{1}{2}\times(b-a)\times h=1$$

$$h=\frac{2}{b-a}$$




For $a \le x \le \dfrac{a+b}{2}$, horizontal distance = $\dfrac{b-a}{2}$

$$m=\frac{2}{b-a}\div\frac{b-a}{2}=\frac{4}{(b-a)^2}$$

$$y=\frac{4x}{(b-a)^2}(x-a)$$

Similarly, for $\dfrac{a+b}{2} \le x \le b,\ y=-\dfrac{4x}{(b-a)^2}(x-b)$

$$p(x)=\begin{cases}\dfrac{4x}{(b-a)^2}(x-a) & \text{for } a\le x\le\dfrac{a+b}{2}\\ -\dfrac{4x}{(b-a)^2}(x-b) & \text{for } \dfrac{a+b}{2}<x\le b\end{cases}$$

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$$E(x)=\int_a^{\frac{a+b}{2}} x\times\left(\frac{4x}{(b-a)^2}(x-a)\right)dx+\int_{\frac{a+b}{2}}^{b} x\times\left(-\frac{4x}{(b-a)^2}(x-b)\right)dx$$

$$=\frac{4}{(b-a)^2}\int_a^{\frac{a+b}{2}} x\times(x-a)dx-\frac{4}{(b-a)^2}\int_{\frac{a+b}{2}}^{b} x(x-b)dx$$

$$=\frac{4}{(b-a)^2}\int_a^{\frac{a+b}{2}} (x^2-ax)dx+\frac{4}{(b-a)^2}\int_b^{\frac{a+b}{2}} (x^2-bx)dx$$

$$=\frac{4}{(b-a)^2}\left[\frac{x^3}{3}-\frac{ax^2}{2}\right]_a^{\frac{a+b}{2}}+\frac{4}{(b-a)^2}\left[\frac{x^3}{3}-\frac{bx^2}{2}\right]_b^{\frac{a+b}{2}}$$

$$=\frac{4}{(b-a)^2}\left(\frac{(a+b)^3}{24}-\frac{a(a+b)^2}{8}-\frac{a^3}{3}+\frac{a^3}{2}+\frac{(a+b)^3}{24}-\frac{b(a+b)^2}{8}-\frac{b^3}{3}+\frac{b^3}{2}\right)$$

$$=\frac{4}{(b-a)^2}\left(\frac{2(a+b)^3}{24}-\frac{(a+b)^2(a+b)}{8}+\frac{a^3}{6}+\frac{b^3}{6}\right)$$

$$=\frac{4}{24(b-a)^2}\left(2(a+b)^3-3(a+b)^3+4(a^3+b^3)\right)$$

$$=\frac{1}{6(b-a)^2}\left(4(a+b)(a^2-ab+b^2)-(a+b)^3\right)$$

$$=\frac{(a+b)}{6(b-a)^2}\left(4(a^2-ab+b^2)-(a+b)^2\right)$$

$$=\frac{(a+b)}{6(b-a)^2}\left(4a^2-4ab+4b^2-a^2-2ab-b^2\right)$$

$$=\frac{(a+b)}{6(b-a)^2}\left(3a^2-6ab+3b^2\right)$$

$$=\frac{(a+b)}{6(b-a)^2}\times 3\left(a-b\right)^2$$

$$=\frac{(a+b)}{2}$$

QED


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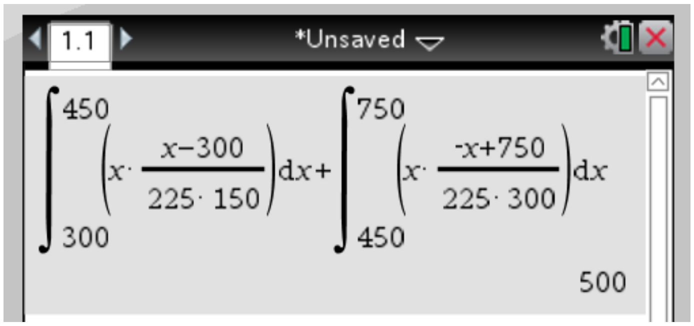
**8** TI-Nspire CAS

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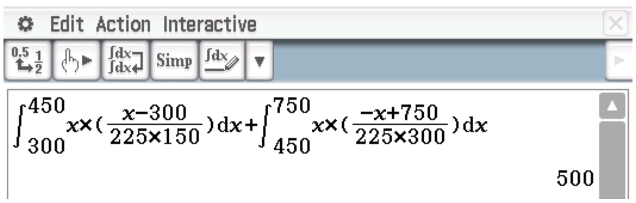
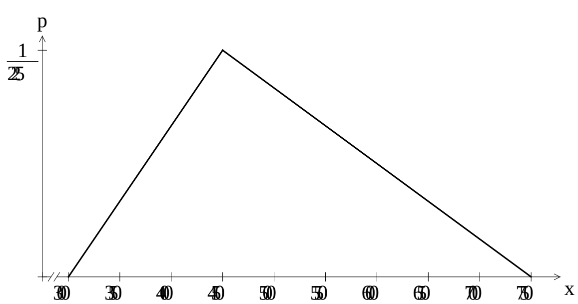



[$300 000, $750 000], width of base is 450 000. Mode is $450 000.

**a** Use [300, 750], $h=\dfrac{1}{225}$




$$p(x)=\begin{cases}\dfrac{x-300}{225\times150} & \text{for } 300\le x\le 450\\[2mm] \dfrac{-(x-750)}{225\times300} & \text{for } 450\le x\le 750\end{cases}$$

$$E(x)=\int_{300}^{450}x\times\left(\frac{x-300}{225\times150}\right)dx+\int_{450}^{750}x\times\left(\frac{-(x-750)}{225\times300}\right)dx$$

$$=500$$

So $E(x)$ = $500 000

**b** 0.5

**c** Now $\int_{300}^{450} \frac{x-300}{225\times150}\,dx = \frac{1}{3}$

Thus, $m$ is such that $\int_{450}^{m} \frac{-(x-750)}{225\times300}\,dx = \frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

Now $\frac{1}{225\times300}\int_{450}^{m}(750-x)\,dx = \frac{1}{225\times300}\left[750x-\frac{x^2}{2}\right]_{450}^{m}$

Thus $\frac{1}{225\times300}\left[750m-\frac{m^2}{2}-750\times450+\frac{450^2}{2}\right] = \frac{1}{6}$

Solving on a calculator, $m = 750 - 150\sqrt{3}$ or $750 + 150\sqrt{3}$

But $750 + 150\sqrt{3}$ is outside the domain, so $m = 750 - 150\sqrt{3} \approx 490.192\ldots$

The median would be about $490 000.

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## Exercise 8.05 Variance and standard deviation

### Concepts and techniques

**1** Given a uniform continuous random variable $X$ defined on the interval [4, 16],

**a** $p(x) = \frac{1}{12}$

**b** by symmetry, $E(x) = 10$

**c** $Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$$\sigma^2 = \int_4^{16} \frac{1}{12}(x-10)^2\,dx$$
$$= 12$$
$$\sigma = \sqrt{12}$$
$$\sigma = 2\sqrt{3}$$

**2** **a** [5, 25], width is 20.

$p(x) = \frac{1}{20}$

By symmetry, $E(x) = 15$

$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$$\sigma^2 = \int_5^{25} \frac{1}{20}(x-15)^2\,dx$$
$$= 33.\overline{3}$$
$$\sigma = \sqrt{33.\overline{3}}$$
$$\sigma \approx 5.77$$

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**b** [0, 50], width is 50

$$p(x) = \frac{1}{50}$$

By symmetry, $E(x) = 25$

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_0^{50} \frac{1}{50}(x-25)^2\,dx$$
$$= 208.\overline{3}$$
$$\sigma = \sqrt{208..\overline{3}}$$
$$\sigma \approx 14.43$$

**c** [0, 20], width is 20

$$p(x) = \frac{1}{20}$$

By symmetry, $E(x) = 10$

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_0^{20} \frac{1}{20}(x-10)^2\,dx$$
$$= 33.\overline{3}$$
$$\sigma = \sqrt{33.\overline{3}}$$
$$\sigma \approx 5.77$$

**d** [80, 120], width is 40.

$$p(x) = \frac{1}{40}$$

By symmetry, $E(x) = 100$

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_{80}^{120} \frac{1}{40}(x-100)^2\,dx$$
$$= 133.\overline{3}$$
$$\sigma = \sqrt{133.\overline{3}}$$
$$\sigma \approx 11.547$$

**e** [0.6, 2.1] width is 1.5.

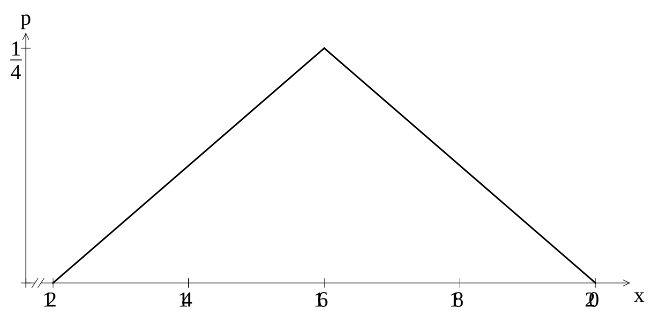
$$p(x)=\frac{2}{3}$$

By symmetry, $E(x)$ = 1.35

$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$$\sigma^2=\int_{0.6}^{2.1}\frac{2}{3}(x-1.35)^2\,dx$$
$$=0.1875$$
$$\sigma=\sqrt{0.1875}$$
$$\sigma\approx 0.433$$

**3** **a** [12, 20], width of base is 8.




$$p(x)=\begin{cases}\frac{x}{16}-\frac{3}{4} & \text{for } 12\le x\le 16\\ -\frac{x}{16}+\frac{5}{4} & \text{for } 16\le x\le 20\end{cases}$$

**b** $E(X)=16$ by symmetry

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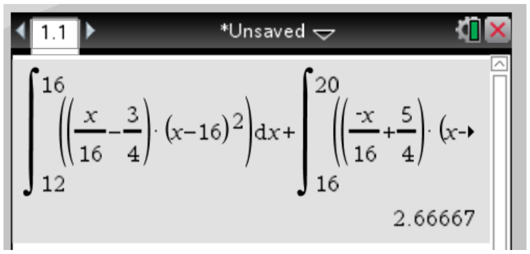
**c** TI-Nspire CAS

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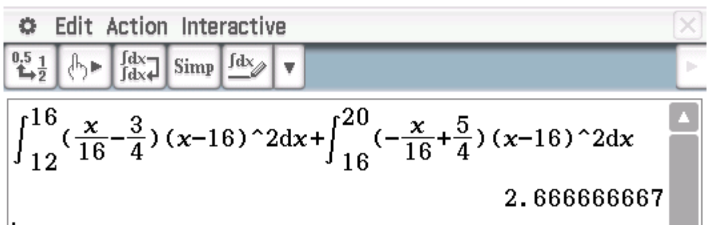



$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_{12}^{16}\left(\frac{x}{16}-\frac{3}{4}\right)(x-16)^2\,dx + \int_{16}^{20}\left(-\frac{x}{16}+\frac{5}{4}\right)(x-16)^2\,dx$$

$$= 1.\overline{3} + 1.\overline{3}$$

$$= 2\frac{2}{3}$$

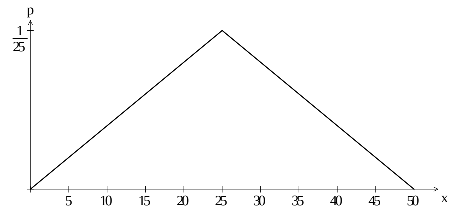
$$\sigma = \sqrt{2\frac{2}{3}}$$

$$\sigma \approx 1.633$$

**4** A continuous random variable $X$ is defined on the interval [0, 50] and has a symmetrical triangular probability density function.

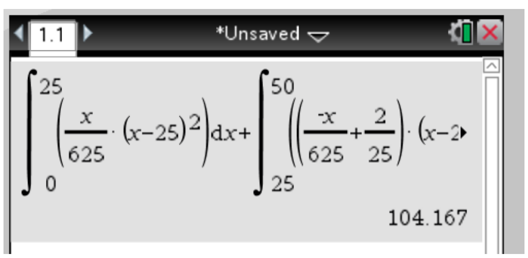
**a**

$$p(x) = \begin{cases} \dfrac{x}{625} & \text{for } 0 \le x \le 25 \\ \dfrac{-(x-50)}{625} & \text{for } 25 \le x \le 50 \end{cases}$$




**b** $E(X) = 25$ by symmetry

**c** TI-Nspire CAS

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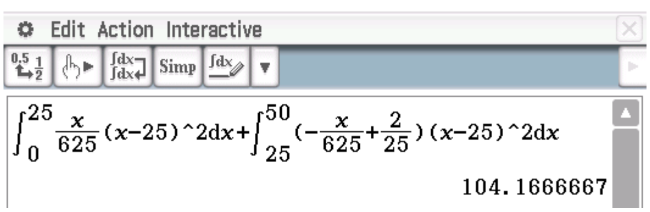


$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_0^{25}\left(\frac{x}{625}\right)(x-25)^2\,dx + \int_{25}^{50}\left(-\frac{x}{625}+\frac{2}{25}\right)(x-25)^2\,dx$$

$$= 52.08\overline{3} + 52.08\overline{3}$$

$$= 104.1\overline{6}$$

$$\sigma = \sqrt{104.1\overline{6}}$$

$$\sigma \approx 10.206$$


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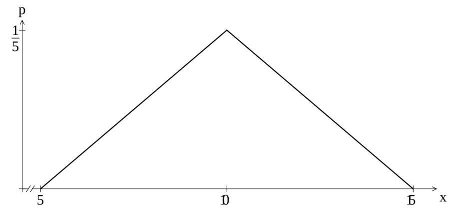
**5** **a** [5, 15]




$$p(x) = \begin{cases} \dfrac{x}{25} - \dfrac{1}{5} & \text{for} \quad 5 \le x \le 10 \\ -\dfrac{x}{25} + \dfrac{3}{5} & \text{for} \quad 10 \le x \le 15 \end{cases}$$

$E(X) = 10$ by symmetry

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_5^{10} \left(\frac{x}{25} - \frac{1}{5}\right)(x-10)^2\,dx + \int_{10}^{15} \left(-\frac{x}{25} + \frac{3}{5}\right)(x-10)^2\,dx$$

$$= 2.08\overline{3} + 2.08\overline{3}$$

$$= 4.1\overline{6}$$

$$\sigma = \sqrt{4.1\overline{6}}$$

$$\sigma \approx 2.04$$

**b** [0, 54]

$$p(x) = \begin{cases} \dfrac{x}{729} & \text{for} \quad 0 \le x \le 27 \\ -\dfrac{x}{729} + \dfrac{2}{27} & \text{for} \quad 27 \le x \le 54 \end{cases}$$

$E(X) = 27$ by symmetry

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_0^{27} \left(\frac{x}{729}\right)(x-27)^2\,dx + \int_{27}^{54} \left(-\frac{x}{729} + \frac{2}{27}\right)(x-27)^2\,dx$$

$$= 60.75 + 60.75$$

$$= 121.5$$

$$\sigma = \sqrt{121.5}$$

$$\sigma \approx 11.02$$

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**c** [6, 60]

$$p(x)=\begin{cases}\dfrac{x}{729}-\dfrac{2}{243} & \text{for} \quad 3\le x\le 33\\ -\dfrac{x}{729}+\dfrac{20}{243} & \text{for} \quad 33\le x\le 60\end{cases}$$

$E(X)=33$ by symmetry

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2 = \int_6^{33}\left(\frac{x}{729}-2\right)(x-33)^2\,dx+\int_{33}^{60}\left(-\frac{x}{729}+\frac{20}{243}\right)(x-33)^2\,dx$$

$$=60.75+60.75$$

$$=121.5$$

$$\sigma=\sqrt{121.5}$$

$$\sigma\approx 11.02$$

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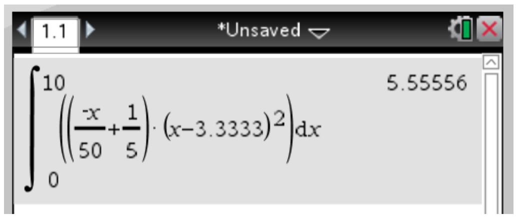
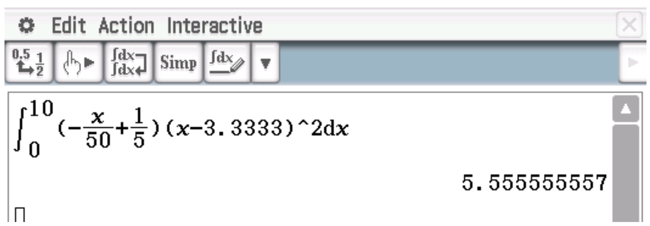
**6** **a** $p(x) = -\dfrac{x}{50} + \dfrac{1}{5}$

**b** $E(X) = \int_0^{10} x \times \left(-\dfrac{x}{50} + \dfrac{1}{5}\right) dx = 3\dfrac{1}{3}$

**c** **TI-Nspire CAS**




**ClassPad**




$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$$\sigma^2 = \int_0^{10} \left(-\frac{x}{50} + \frac{1}{5}\right)(x - 3.\overline{3})^2\,dx$$

$$= 5.\overline{5}$$

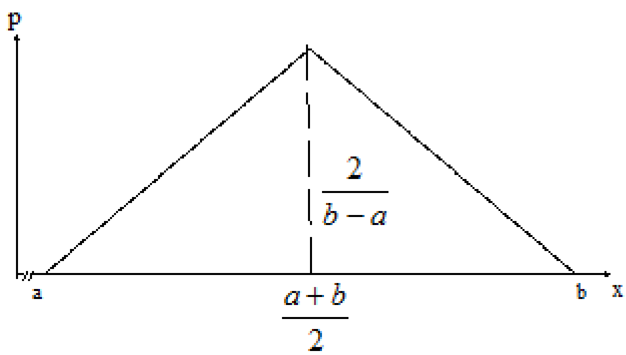
$$\sigma = \sqrt{5.\overline{5}}$$

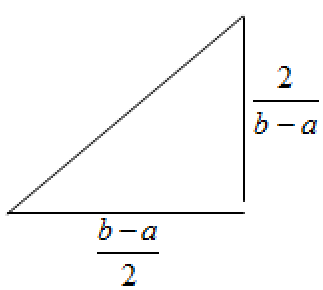
$$\sigma \approx 2.357$$

**7** Width of base is $b - a$, Midpoint: $\frac{a+b}{2}$, Area of triangle = 1.

$$\frac{1}{2}\times(b-a)\times h = 1$$

$$h = \frac{2}{b-a}$$




For $a \le x \le \frac{4}{(b-a)^2}(x-a)\ \frac{a+b}{2}$

$$m = \frac{2}{b-a} \div \frac{b-a}{2} = \frac{4}{(b-a)^2}$$

For $\frac{a+b}{2} \le x \le b$, $m = -\frac{4}{(b-a)^2}$

$$p(x) = \begin{cases} \frac{4}{(b-a)^2}(x-a) & \text{for } a \le x \le \frac{a+b}{2} \\ -\frac{4}{(b-a)^2}(x-b) & \text{for } \frac{a+b}{2} \le x \le b \end{cases}$$

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Also, $u = \dfrac{a+b}{2}$

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$= \int_a^{\frac{a+b}{2}} \frac{4}{(b-a)^2}(x-a)\left(x-\frac{a+b}{2}\right)^2 dx + \int_{\frac{a+b}{2}}^{b} -\frac{4}{(b-a)^2}(x-b)\left(x-\frac{a+b}{2}\right)^2 dx$$

$$= \frac{4}{(b-a)^2}\int_a^{\frac{a+b}{2}} (x-a)\left(\frac{2x-a-b}{2}\right)^2 dx - \frac{4}{(b-a)^2}\int_{\frac{a+b}{2}}^{b}(x-b)\left(\frac{2x-a-b}{2}\right)^2 dx$$

$$= \frac{1}{(b-a)^2}\int_a^{\frac{a+b}{2}} (x-a)(2x-a-b)^2\,dx + \frac{1}{(b-a)^2}\int_b^{\frac{a+b}{2}}(x-b)(2x-a-b)^2\,dx$$

$$= \frac{1}{(b-a)^2}\left[\int_a^{\frac{a+b}{2}} (x-a)(4x^2+a^2+b^2-4ax-4bx+2ab)\,dx \right.$$
$$\left. + \int_b^{\frac{a+b}{2}} (x-b)(4x^2+a^2+b^2-4ax-4bx+2ab)\,dx\right]$$

$$= \frac{1}{(b-a)^2}\left[\int_a^{\frac{a+b}{2}} (4x^3-8ax^2-4bx^2+5a^2x+6abx+b^2x-a^3-2a^2b-ab^2)\,dx \right.$$
$$\left. + \int_b^{\frac{a+b}{2}} (4x^3-4ax^2-8bx^2+a^2x+6abx+5b^2x-a^2b-2ab^2-b^3)\,dx\right]$$

$$= \frac{1}{(b-a)^2}\left(\left[x^4-\frac{8a+4b}{3}x^3+\frac{5a^2+6ab+b^2}{2}x^2-(a^3+2a^2b+ab^2)x\right]_a^{\frac{a+b}{2}}\right.$$
$$\left. + \left[x^4-\frac{4a+8b}{3}x^3+\frac{a^2+6ab+5b^2}{2}x^2-(a^2b+2ab^2+b^3)x\right]_b^{\frac{a+b}{2}}\right)$$

$$= \frac{1}{(b-a)^2}\left\{\left(\frac{a+b}{2}\right)^4-\frac{8a+4b}{3}\left(\frac{a+b}{2}\right)^3+\frac{5a^2+6ab+b^2}{2}\left(\frac{a+b}{2}\right)^2\right.$$
$$-(a^3+2a^2b+ab^2)\left(\frac{a+b}{2}\right)-a^4+\frac{8a+4b}{3}a^3-\frac{5a^2+6ab+b^2}{2}a^2$$
$$+(a^3+2a^2b+ab^2)a+\left(\frac{a+b}{2}\right)^4-\frac{4a+8b}{3}\left(\frac{a+b}{2}\right)^3+\frac{a^2+6ab+5b^2}{2}\left(\frac{a+b}{2}\right)^2$$
$$\left.-(a^2b+2ab^2+b^3)\left(\frac{a+b}{2}\right)-b^4+\frac{4a+8b}{3}b^3-\frac{a^2+6ab+5b^2}{2}b^2+(a^2b+2ab^2+b^3)b\right\}$$

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$$= \frac{1}{(b-a)^2}\left\{2\frac{(a+b)^4}{16} - \frac{(12a+12b)(a+b)^3}{3\times 8} + \frac{(6a^2+12ab+6b^2)(a+b)^2}{8}\right.$$

$$- \frac{(a^3+3a^2b+3ab^2+b^3)(a+b)}{2} - a^4 + \frac{8a+4b}{3}a^3 - \frac{5a^2+6ab+b^2}{2}a^2$$

$$\left. +(a^3+2a^2b+ab^2)a - b^4 + \frac{4a+8b}{3}b^3 - \frac{a^2+6ab+5b^2}{2}b^2 + (a^2b+2ab^2+b^3)b\right\}$$

$$= \frac{1}{24(b-a)^2}\left\{3(a+b)^4 - 12(a+b)(a+b)^3 + 3\times 6(a^2+2ab+b^2)(a+b)^2\right.$$

$$-12\times(a^3+3a^2b+3ab^2+b^3)(a+b) - 24a^4 + 8\times 4(2a+b)a^3$$

$$-12(5a^2+6ab+b^2)a^2 + 24(a^3+2a^2b+ab^2)a - 24b^4 + 8\times 4(a+2b)b^3$$

$$\left. -12(a^2+6ab+5b^2)b^2 + 24(a^2b+2ab^2+b^3)b\right\}$$

$$= \frac{1}{24(b-a)^2}\left\{3(a+b)^4 - 12(a+b)(a+b)^3 + 18(a+b)^2(a+b)^2 - 12(a+b)^3(a+b)\right.$$

$$-24a^4 + 64a^4 + 32a^3b - 60a^4 - 72a^3b - 12a^2b^2 + 24a^4 + 48a^3b + 24a^2b^2$$

$$\left. -24b^4 + 32ab^3 + 64b^4 - 12a^2b^2 - 72ab^3 - 60b^4 + 24a^2b^2 + 48ab^3 + 24b^4\right\}$$

$$= \frac{1}{24(b-a)^2}\left\{-3(a+b)^4 + 4a^4 + 8a^3b + 12a^2b^2 + 8ab^3 + 4b^4 + 12a^2b^2\right\}$$

$$= \frac{1}{24(b-a)^2}\left(-3a^4 - 12a^3b - 18a^2b^2 - 12ab^3 - 3b^4 + 4a^4 + 8a^3b + 24a^2b^2 + 8ab^3 + 4b^4\right)$$

$$= \frac{1}{24(b-a)^2}\left(a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4\right)$$

$$= \frac{1}{24(b-a)^2}\left(a-b\right)^4 = \frac{(b-a)^2}{24} \qquad \text{QED}$$

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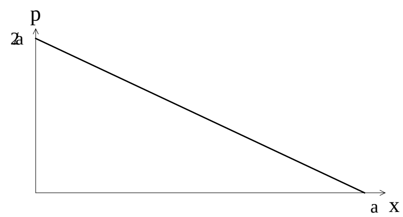
**8**




$$p(x) = -\frac{2}{a^2}(x-a)$$

$$E(X) = \int_0^a x \times p(x)dx$$

$$= \int_0^a x \times -\frac{2}{a^2}(x-a)dx$$

$$= \frac{2}{a^2}\int_0^a (ax - x^2)dx$$

$$= \frac{2}{a^2}\left[\frac{ax^2}{2} - \frac{x^3}{3}\right]_0^a$$

$$\mu = \frac{2}{a^2}\left(\frac{a^3}{2} - \frac{a^3}{3}\right) = \frac{2}{a^2} \times \frac{a^3}{6} = \frac{a}{3}$$

$$Var(X) = \int_0^a p(x)(x-\mu)^2 dx$$

$$= \int_0^a -\frac{2}{a^2}(x-a)\left(x - \frac{a}{3}\right)^2 dx$$

$$= \frac{2}{9a^2}\int_0^a (a-x)(3x-a)^2 dx$$

$$= \frac{2}{9a^2}\int_0^a (a-x)(9x^2 - 6ax + a^2)dx$$

$$= \frac{2}{9a^2}\int_0^a (9ax^2 - 6a^2x + a^3 - 9x^3 + 6ax^2 - a^2x)dx$$

$$= \frac{2}{9a^2}\left[3ax^3 - 3a^2x^2 + a^3x - \frac{9}{4}x^4 + 2ax^3 - \frac{1}{2}a^2x^2\right]_0^a$$

$$= \frac{2}{9a^2}\left(3a^4 - 3a^4 + a^4 - \frac{9}{4}a^4 + 2a^4 - \frac{1}{2}a^4\right)$$

$$= \frac{2}{9a^2} \times \frac{a^4}{4} = \frac{a^2}{18}$$

Thus the variance is given by $\frac{a^2}{18}$.

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## Exercise 8.06 Linear changes of scale and origin

### Concepts and techniques

**1** **a** $p(x) = \frac{1}{100}$

$E(x) = 50$ from symmetry

$Var(X) = \int_a^b p(x)(x - \mu)^2 \, dx$

$Var(X) = \int_0^{100} \left(\frac{1}{100}\right)(x - 50)^2 \, dx$

$= 833.\overline{3}$

$\sigma_x = \sqrt{Var(X)}$

$= 28.87$

**b** [5, 205]

**c** $Y = 2X + 5$

$E(x) = 50 \Rightarrow E(y) = 2 \times 50 + 5 = 105$

$Var(X) = \sigma_x^2 = 833.\overline{3}$

$Var(Y) = \sigma_y^2 = 2^2 \sigma_x^2 = 3333.\overline{3}$

$\sigma_x = 28.87$

$\sigma_y = 2 \times 28.87 = 57.74$

**d** $E(Y) = 2E(X) + 5$

$Var(Y) = 4Var(X)$

$SD(Y) = 2SD(X)$

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**2** **a** $p(x) = \dfrac{1}{20}$ on the interval [30, 50]

$E(x) = 40$ from symmetry

$Var\left(X\right) = \int_a^b p(x)(x-\mu)^2\,dx$

$Var\left(X\right) = \int_{30}^{50}\left(\dfrac{1}{20}\right)\left(x-40\right)^2\,dx$

$= 33.\overline{3}$

$\sigma_x = \sqrt{Var\left(X\right)}$

$\sigma_x = 5.77$

**b** [148, 248]

**c** $Y = 5X - 2$

$E(x) = 50 \Rightarrow E(y) = 5\times 40 - 2 = 198$

$Var\left(X\right) = \sigma_x^2 = 33.\overline{3}$

$Var\left(Y\right) = \sigma_y^2 = 5^2\sigma_x^2 = 833.\overline{3}$

$\sigma_x = 5.77$

$\sigma_y = 5\times 5.77 = 28.87$

**d** $E(Y) = 5E(X) - 2$

$Var(Y) = 25Var(X)$

$SD(Y) = 5SD(X)$

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**3** **a** $p(x) = \frac{1}{40}$ on the interval [20, 60]

$E(x) = 40$ from symmetry

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$Var(X) = \int_{20}^{60}\left(\frac{1}{40}\right)(x-40)^2\,dx$$

$$= 133.\overline{3}$$

$$\sigma_x = \sqrt{Var(X)}$$

$$= 11.547$$

**b** [8, 16]

**c** $Y = 0.2X + 4$

$$E(x) = 50 \quad \Rightarrow E(y) = 0.2 \times 40 + 4 = 12$$

$$Var(X) = \sigma_x^2 = 133.\overline{3}$$

$$Var(Y) = \sigma_y^2 = 0.2^2\sigma_x^2 = 5.\overline{3}$$

$$\sigma_y = 2.31$$

**d** $E(Y) = 0.2E(X) + 4$

$Var(Y) = 0.2^2\ Var(X)$

$SD(Y) = 0.2SD(X)$

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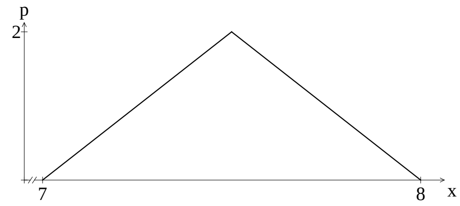
**4** A continuous random variable $X$ is defined on the interval [7, 8] and has a symmetrical triangular probability density function.




$$p(x) = \begin{cases} 4x - 28 & \text{for } 7 \le x \le 7.5 \\ -4x + 32 & \text{for } 7.5 \le x \le 8 \end{cases}$$

**a** $E(x) = 7.5$ from symmetry

$$Var(X) = \int_a^b p(x)(x - \mu)^2\, dx$$

$$Var(X) = \int_7^{\frac{15}{2}} (4x - 28)\left(x - \frac{15}{2}\right)^2 dx + \int_{\frac{15}{2}}^{8} (-4x + 32)\left(x - \frac{15}{2}\right)^2 dx$$

$$= \frac{1}{24}$$

$$\sigma_x = \sqrt{Var(X)}$$

$$\sigma_x = \frac{\sqrt{6}}{12} \approx 0.2041...$$

**b** [110, 130]

**c** $Y = 20X - 30$

$$E(x) = 7.5 \quad \Rightarrow E(y) = 20 \times 7.5 - 39 = 120$$

$$Var(Y) = \sigma_y^2 = 20^2 \sigma_x^2 = \frac{400}{24} = 16\tfrac{2}{3}$$

$$\sigma_y = \frac{5\sqrt{6}}{3} \approx 4.082$$

**d** $E(Y) = 20E(X) - 30$

$Var(Y) = 20^2\ Var(X)$

$SD(Y) = 20SD(X)$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**5** [0, 50]

**a** $$p(x)=\begin{cases}\dfrac{x}{625} & \text{for } 0\le x\le 25\\ -\dfrac{x}{625}+\dfrac{2}{25} & \text{for } 25\le x\le 50\end{cases}$$

$E(X) = 25$ by symmetry

$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$$\sigma^2 = \int_0^{25}\left(\frac{x}{625}\right)(x-25)^2\,dx + \int_{25}^{50}\left(-\frac{x}{625}+\frac{2}{25}\right)(x-25)^2\,dx$$
$$= 52.08\overline{3} + 52.08\overline{3}$$
$$= 104.1\overline{6}$$
$$\sigma = \sqrt{104.1\overline{6}}$$
$$\approx 10.21$$

**b** $Y = 3X - 5$

[–5, 145]

**c** $E(x) = 25 \quad \Rightarrow E(y) = 3\times 25 - 5 = 70$

$Var\left(X\right) = \sigma_x^2 = 104.1\overline{6}$

$Var\left(Y\right) = \sigma_y^2 = 3^2\sigma_x^2 = 937.5$

$\sigma_y = 30.62$

**d** $E(Y) = 3E(X) - 5$

$Var(Y) = 3^2\ Var(X)$

$SD(Y) = 3SD(X)$


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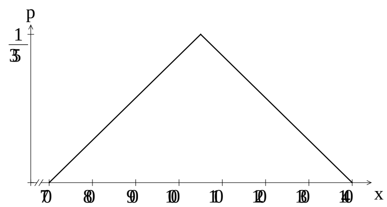
**6** [70, 140]

**a** $$p(x)=\begin{cases}\dfrac{x}{1225}-\dfrac{2}{35} & \text{for } 70\le x\le 105\\ -\dfrac{x}{1225}+\dfrac{4}{35} & \text{for } 105\le x\le 140\end{cases}$$



$E(X)=105$ by symmetry

$$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$$

$$\sigma^2=\int_{70}^{105}\left(\frac{x}{1225}-\frac{2}{35}\right)(x-105)^2\,dx+\int_{105}^{140}\left(-\frac{x}{1225}+\frac{4}{35}\right)(x-105)^2\,dx$$

$$=102.08\overline{3}+102.08\overline{3}$$

$$=204.1\overline{6}$$

$$\sigma=\sqrt{204.1\overline{6}}$$

$$\approx 14.29$$

**b** $Y = 0.1X + 2.5$, [9.5, 16.5]

**c** $E(x)=105 \quad \Rightarrow E(y)=0.1\times 105+2.5=13$

$Var\left(X\right)=\sigma_x^2=204.1\overline{6}$

$Var\left(Y\right)=\sigma_y^2=0.1^2\sigma_x^2=2.042$

$\sigma_y=1.429$

**d** $E(Y) = 0.1E(X) + 2.5$

$Var(Y) = 0.1^2\ Var(X)$

$SD(Y) = 0.1SD(X)$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**7** $X$, [40, 90], $\mu = 55$ and $\sigma = 5$.

$Y = 3X + 8$

$E(Y) = 3(X) + 8$

$= 3(55) + 8$

$E(Y) = 173$

$Var(Y) = 3^2\ Var(X)$

$= 9 \times 5^2$

$Var(Y) = 225$

$SD(Y) = 3 \times SD(X)$

$SD(Y) = 15$

**8** $X$, [4, 19], $E(X) = 14$ and $Var(X) = 8$.

$Y = 4X - 10$.

$E(Y) = 4(X) - 10$

$= 4(14) - 19$

$E(Y) = 46$

$SD(Y) = 4 \times SD(X)$

$SD(X) = \sqrt{8}$

$SD(Y) = 4\sqrt{8} = 8\sqrt{2}$

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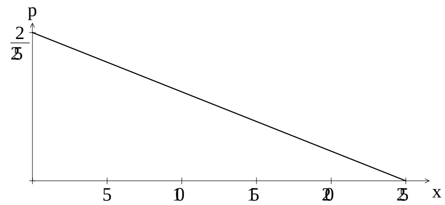
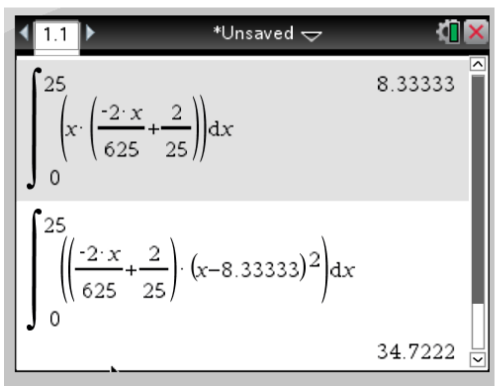
## Reasoning and communication

**9** $X$, [0, 25], maximum value at $x = 0$, $Y = 8X + 200$.




**a** $p(x) = \dfrac{-2x}{625} + \dfrac{2}{25}$

**b** TI-Nspire CAS

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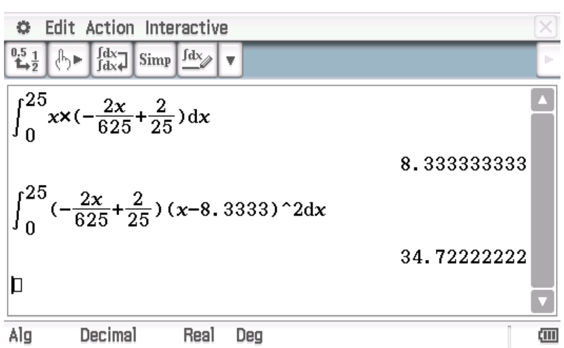


$$E(X) = \int_0^{25} x \times \left(-\frac{2x}{625} + \frac{2}{25}\right) dx = 8.\overline{3}$$

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$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$\sigma^2 = \int_0^{25}\left(-\frac{2x}{625}+\frac{2}{25}\right)\left(x-8.\overline{3}\right)^2\,dx$

$= 34.7\overline{2}$

$\sigma = 5.89$

**c** $Y = 8X + 200.$

[200, 400]

**d** $p(y) = -0.000\,05y + 0.02,\ 200 \le y \le 400$

$Y = 8X + 200$

**e** $E(Y) = 8(X) + 200$

$= 8(8.\overline{3}) + 200$

$E(Y) = 266.\overline{6}$

$Var(Y) = 8 \times Var(X)$

$Var(Y) = 2222.\overline{2}$

$SD(Y) = 8 \times SD(X)$

$SD(X) = 5.892\,556\,51$

$SD(Y) = 47.14$

**f** $E(Y) = 8E(X) + 200$

$Var(Y) = 8^2\ Var(X)$

$SD(Y) = 8SD(X)$

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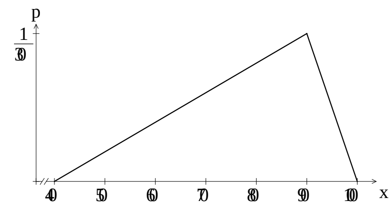
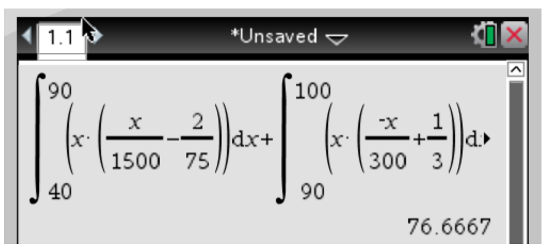
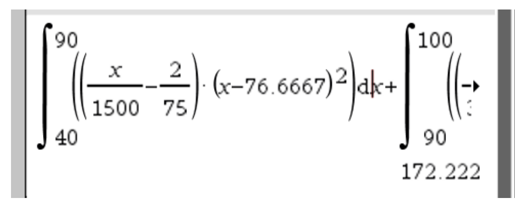
**10** $X$, [40, 100]. maximum value at $x = 90$, $Y = 2X - 15$.




**a** $$p(x) = \begin{cases} \dfrac{x}{1500} - \dfrac{2}{75} & \text{for } 40 \le x \le 90 \\ -\dfrac{x}{300} + \dfrac{1}{3} & \text{for } 90 \le x \le 100 \end{cases}$$

**b** TI-Nspire CAS

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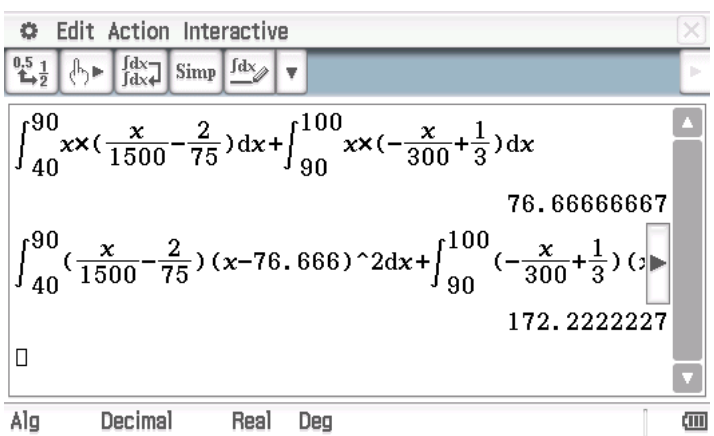


$$E(X) = \int_{40}^{90} x \times \left(\frac{x}{1500} - \frac{2}{75}\right) dx + \int_{90}^{100} x \times \left(-\frac{x}{300} + \frac{1}{3}\right) dx$$

$$= 61.\bar{1} + 15.\bar{5}$$

$$= 76.\bar{6}$$


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$Var(X) = \int_a^b p(x)(x-\mu)^2\,dx$

$$\sigma^2 = \int_{40}^{90}\left(\frac{x}{1500}-\frac{2}{75}\right)(x-76.\overline{6})^2\,dx + \int_{90}^{100}\left(-\frac{x}{300}+\frac{1}{3}\right)(x-76.\overline{6})^2\,dx$$

$$= 125 + 47.\overline{2}$$

$$\sigma^2 = 172.\overline{2}$$

$$\sigma = 13.12$$

**c** $Y = 2X - 15.$

[65, 185]

**d**

$$p(x) = \begin{cases} \dfrac{x}{6000} - \dfrac{13}{1200} & \text{for } 65 \le x \le 165 \\ -\dfrac{x}{1200} + \dfrac{37}{240} & \text{for } 165 \le x \le 185 \end{cases}$$

**e** $Y = 2X - 15$

$E(Y) = 2[E(X)] - 15$

$= 2(76.\overline{6}) - 15$

$E(Y) = 138.\overline{3}$

$Var(Y) = 2^2 \times Var(X)$

$Var(Y) = 4 \times 172.\overline{2} = 688.\overline{8}$

$SD(Y) = 2 \times SD(X)$

$SD(Y) = 26.24$

**f** $E(Y) = 2E(X) - 15$

$Var(Y) = 2^2\, Var(X)$

$SD(Y) = 2SD(X)$

 ISBN 9780170254663 

## Exercise 8.07 The normal distribution and standard normal distribution

### Concepts and techniques

**1** **a** $\mu = 28.5$ and $\sigma = 3.2$

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{3.2\times 2.5066}e^{-\frac{(x-28.5)^2}{2\times(3.2)^2}} = 0.1247e^{-\frac{(x-28.5)^2}{20.48}}$$

**b** $\mu = 28.5$ and $\sigma = 5.7$

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{5.7\times 2.5066}e^{-\frac{(x-28.5)^2}{2\times(5.7)^2}} = 0.07e^{-0.0154(x-28.5)^2}$$

**c** $\mu = 48.6$ and $\sigma = 5.7$

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.07e^{-0.0154(x-48.56)^2}$$

**d** $\mu = 246$ and $\sigma = 78$

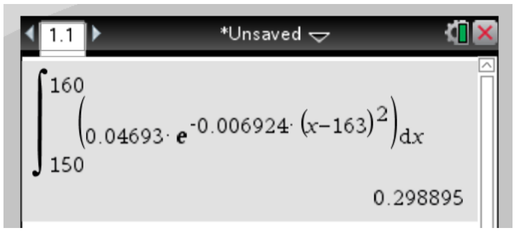
$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.00511e^{-0.00008224(x-246)^2}$$

**e** $\mu = 0.07$ and $\sigma = 0.0024$

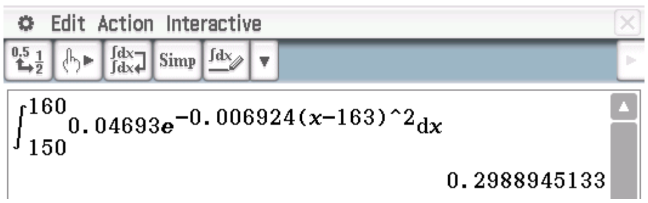
$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 166.226e^{-86805.6(x-0.07)^2}$$

 ISBN 9780170254663 

**2** **a** TI-Nspire CAS




ClassPad




μ = 163 and σ = 8.5, 150 to 160

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.04693e^{-0.006924(x-163)^2}$$

$$\int_{150}^{160} 0.04693e^{-0.006924(x-163)^2}\,dx = 0.299$$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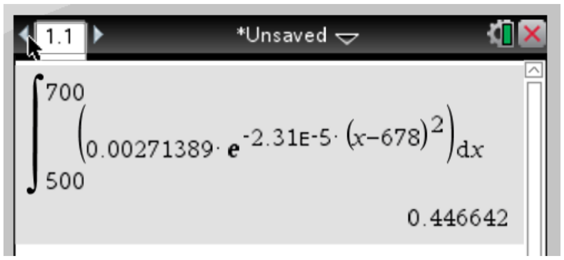
**b** TI-Nspire CAS

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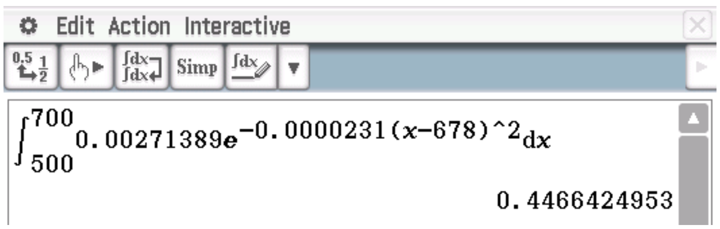
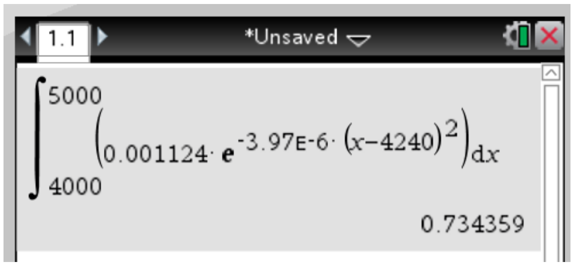



$\mu = 678$ and $\sigma = 147$, 500 to 700

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.00271389e^{-0.0000231(x-678)^2}$$

$$\int_{500}^{700} 0.00271389e^{-0.0000231(x-678)^2}\,dx = 0.4465$$

**c** TI-Nspire CAS

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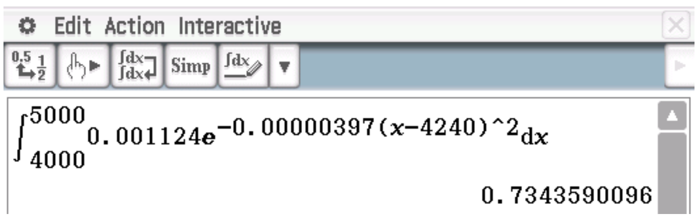


$\mu = 4240$ and $\sigma = 355$, 4000 to 5000

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.001124e^{-0.00000397(x-4240)^2}$$

$$\int_{4000}^{5000} 0.001124e^{-0.00000397(x-4240)^2}\,dx = 0.734$$

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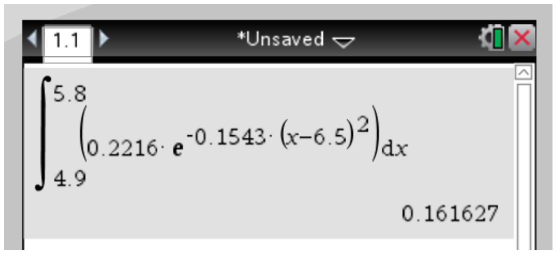
**d** TI-Nspire CAS

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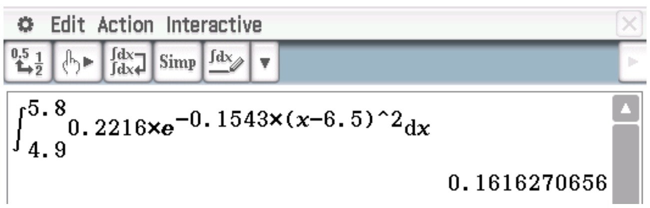
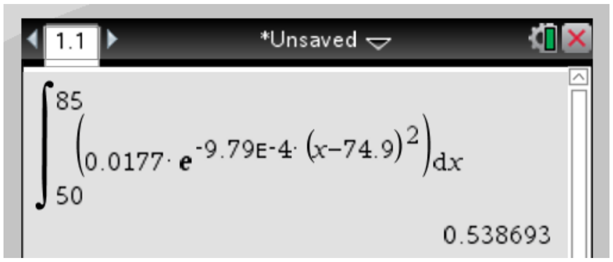



$\mu = 6.5$ and $\sigma = 1.8$, 4.9 to 5.8

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.2216e^{-0.1543(x-6.5)^2}$$

$$\int_{150}^{160} 0.2216e^{-0.1543(x-6.5)^2}\,dx = 0.162$$

**e** TI-Nspire CAS

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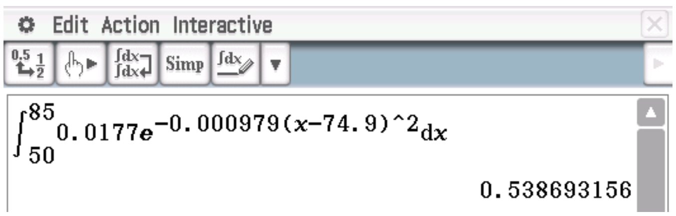
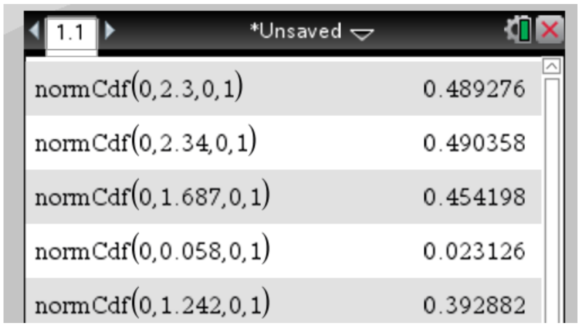



$\mu = 74.9$ and $\sigma = 22.6$, 50 to 85

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.0177e^{-0.000979(x-74.9)^2}$$

$$\int_{150}^{160} 0.0177e^{-0.000979(x-74.9)^2}\,dx = 0.537$$

**3** TI-Nspire CAS

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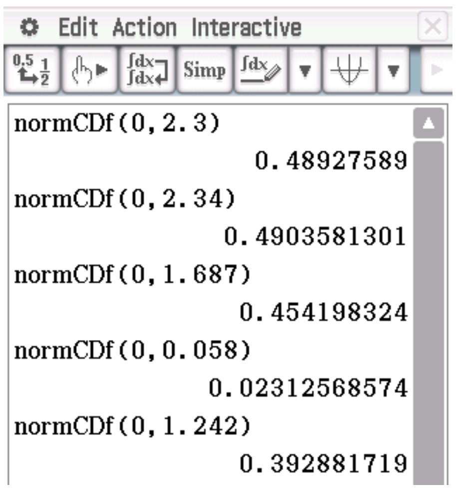


**a** $0 \le Z \le 2.3$

Area is 0.489

**b** $0 \le Z \le 2.34$

Area is 0.490

**c** $0 \le Z \le 1.687$

Area is 0.454

**d** $0 \le Z \le 0.058$

Area is 0.023

**e** $0 \le Z \le 1.242$

Area is 0.393

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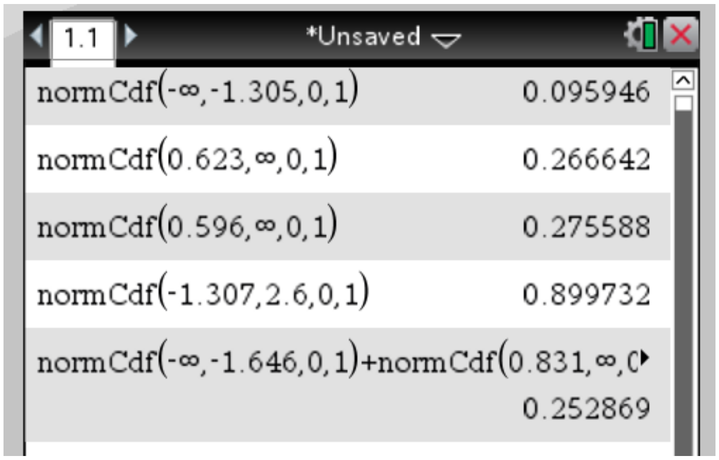
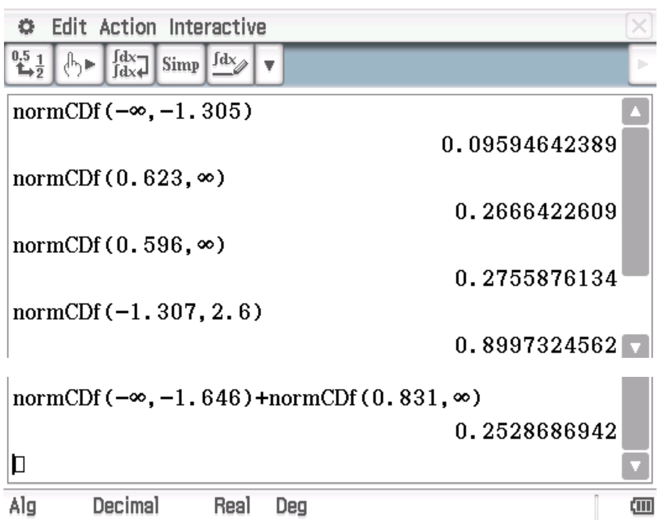
**4** TI-Nspire CAS




ClassPad




**a** $P(Z < -1.305) = 0.096$

**b** $P(Z > 0.623) = 0.267$

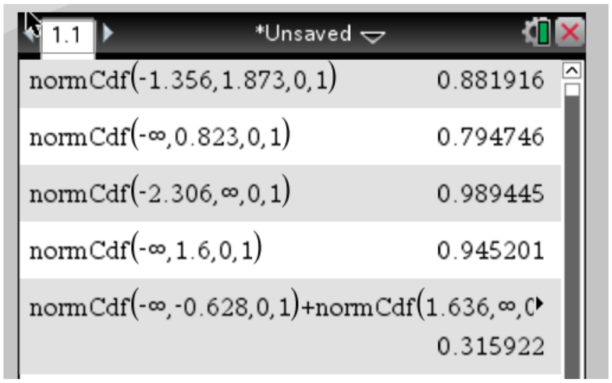
**c** $P(Z > 0.596) = 0.276$

**d** $P(-1.307 < Z < 2.6) = 0.8997$

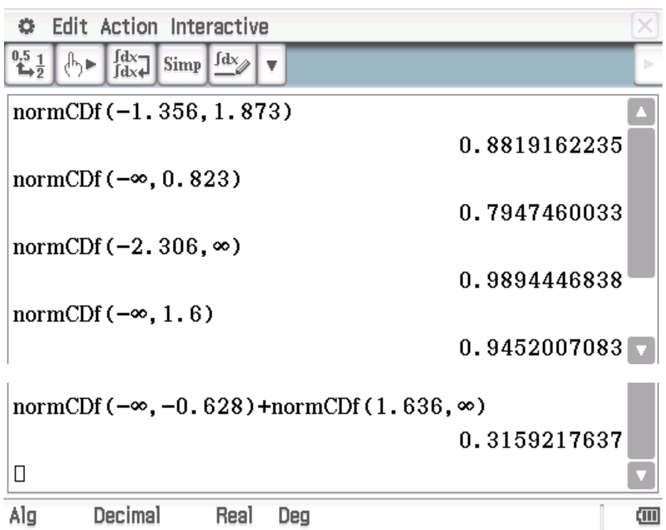
**e** $P(Z < -1.646 \text{ or } Z > 0.831) = 0.04988 + 0.20299 = 0.252$

5 **TI-Nspire CAS**




**ClassPad**




**a** $P(-1.356 < Z < 1.873) = 0.8819$

**b** $P(Z < 0.823) = 0.7947$

**c** $P(Z > -2.306) = 0.989$

**d** $P(Z < 1.6) = 0.945$

**e** $P(Z < -0.628 \text{ or } Z > 1.636) = 0.316$

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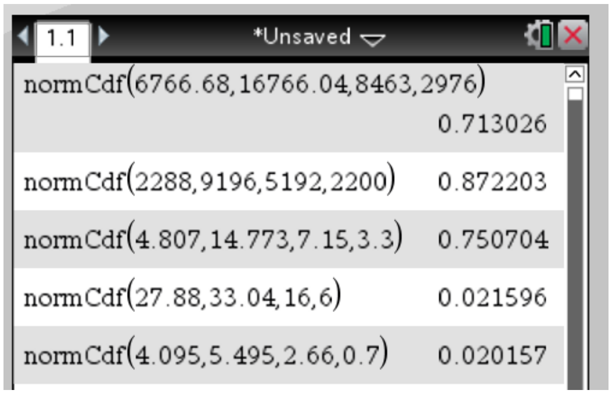
**6** TI-Nspire CAS

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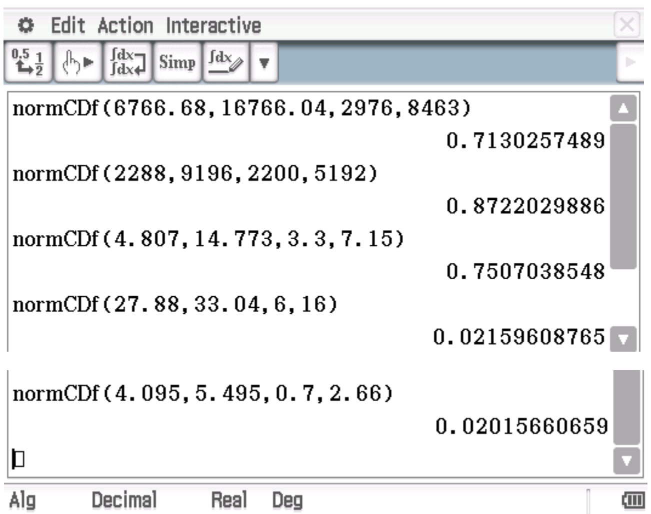



**a** $\mu = 8463$, $\sigma = 2976$

$P(6766.68 < X < 16\,766.04) = 0.713$

**b** $\mu = 5192$, $\sigma = 2200$

$P(2288 < X < 9196) = 0.872$

**c** $\mu = 7.15$, $\sigma = 3.3$

$P(4.807 < X < 14.773) = 0.751$

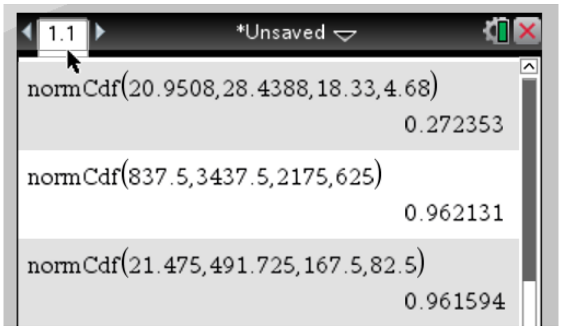
**d** $\mu = 16$, $\sigma = 6$

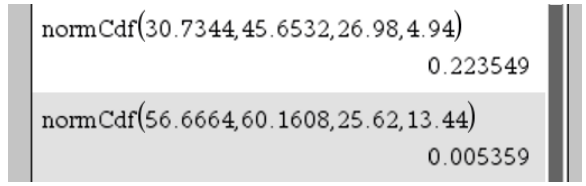
$P(27.88 < X < 33.04) = 0.022$

**e** $\mu = 2.66$, $\sigma = 0.7$

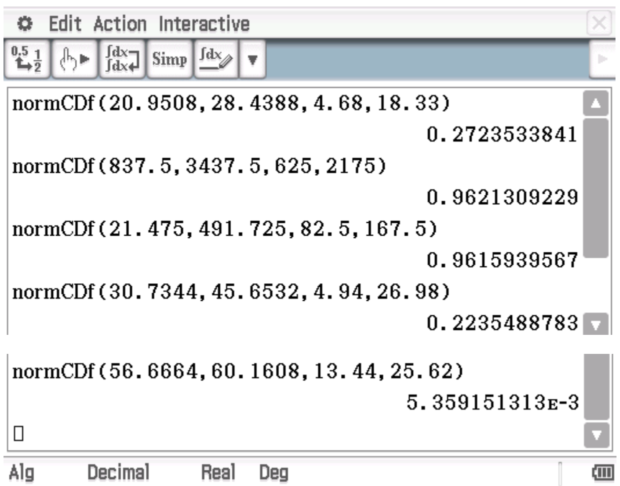
$P(4.095 < X < 5.495) = 0.020$

**7** TI-Nspire CAS




ClassPad




**a** $\mu = 18.33$, $\sigma = 4.68$

$P(20.9508 < X < 28.4388) = 0.272$

**b** $\mu = 2175$, $\sigma = 625$

$P(837.5 < X < 3437.5) = 0.962$

**c** $\mu = 167.5$, $\sigma = 82.5$

$P(21.475 < X < 491.725) = 0.9615$

**d** $\mu = 26.98$, $\sigma = 4.94$

$P(30.7344 < X < 45.6532) = 0.224$

**e** $\mu = 25.62$, $\sigma = 13.44$

$P(56.6664 < X < 60.1608) = 0.005$

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## Reasoning and communication

**8** $\mu$ = 50 000 km and $\sigma$ = 6150 km.

**a** $P(X > 60\ 000 \text{ km}) = 0.051\ 973$

**b** $P(45\ 000 < X < 55\ 000 \text{ km}) = 0.5838$

**c** $P(X < 42\ 000) = 0.096\ 66$, i.e. 9.7%

**9** $\mu$ = 90 000 km and $\sigma$ = 8300 km.

$P(X > 100\ 000 \text{ km}) = 0.1141$

11.4% can be expected to last more than 100 000 km.

**10** $\mu$ = US\$1.03 and $\sigma$ = US\$0.027

$P(X < \text{US\$1}) = 0.133\ 26$

**11** $\mu$ = 4 cm and $\sigma$ = 1.2 cm.

$P(X > 5) = 0.2023$

**12** $\mu$ = 6 m and $\sigma$ = 1.5 m.

$P(X < 3) = 0.022\ 75$

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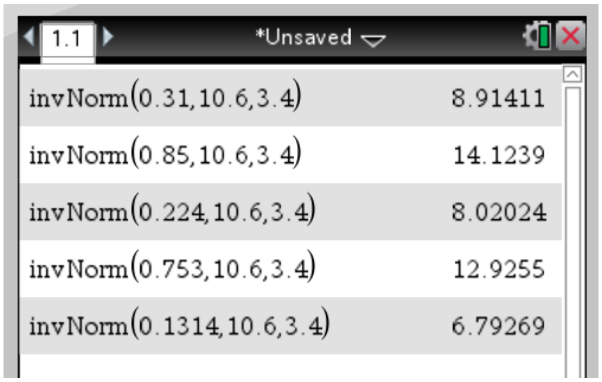
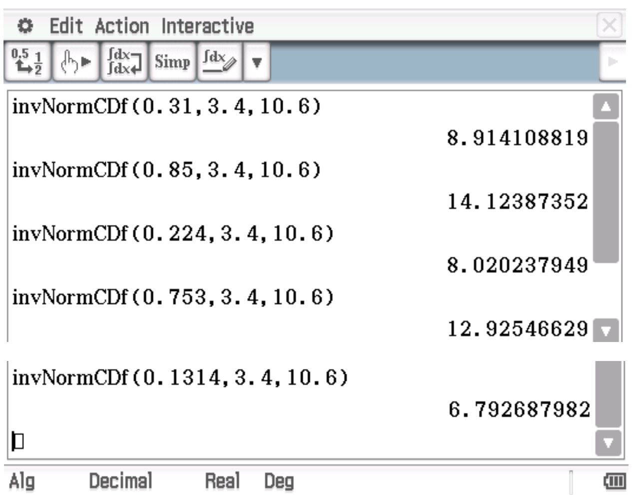
## Exercise 8.08 Standardisation and quantiles

### Concepts and techniques

**1** TI-Nspire CAS




ClassPad




**a** $t_{0.31} = 8.914$

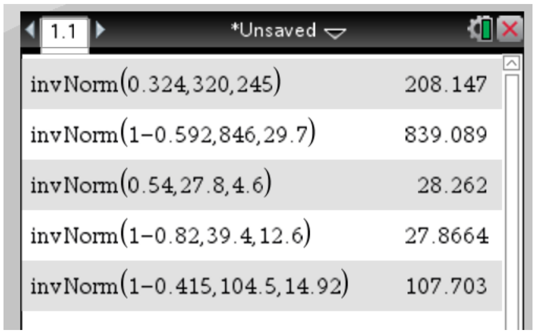
**b** $t_{0.85} = 14.12$

**c** $t_{0.224} = 8.02$

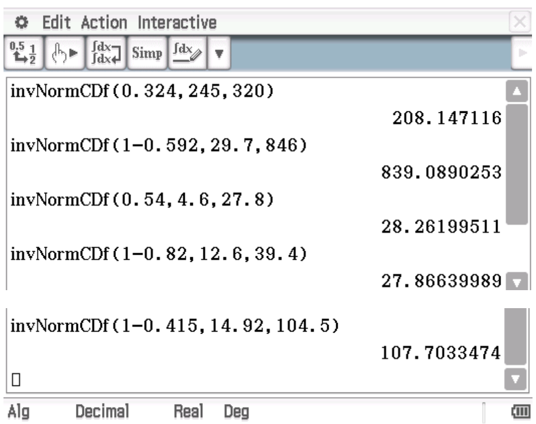
**d** $t_{0.753} = 12.93$

**e** $t_{0.1314} = 6.793$

**2** TI-Nspire CAS




ClassPad




**a** $\mu = 320$, $\sigma = 245$
$P(x < a) = 0.324$
$a = 208.15$

**b** $\mu = 846$, $\sigma = 29.7$
$P(x > a) = 0.592$
$a = 839.09$

**c** $\mu = 27.8$, $\sigma = 4.6$
$P(x \leq a) = 0.54$
$a = 28.26$

**d** $\mu = 39.4$, $\sigma = 12.6$
$P(x > a) = 0.82$
$a = 27.87$

**e** $\mu = 104.5$, $\sigma = 14.92$
$P(x > a) = 0.415$
$a = 107.7$

**3** $X$, $\mu = 74$ and $\sigma = 8.2$.

**a** $P(-\infty < X \le 64) = 0.1113$

**b** $P(64 \le X < a) = 0.6$

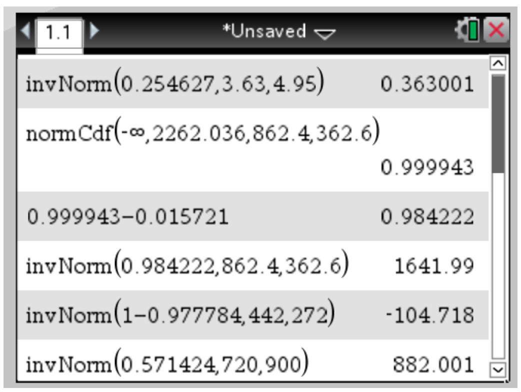
$P(64 \le X < a) = P(X < a) - P(X \le 64)$

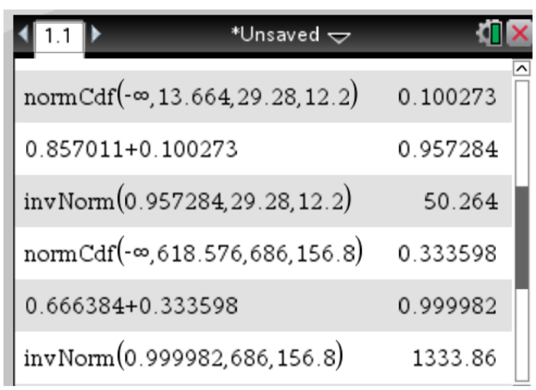
$0.6 = P(X < a) - 0.1113$

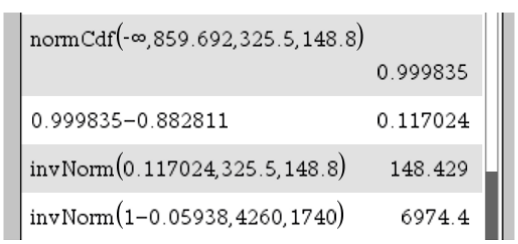
$P(X < a) = 0.7113$

$a = 78.57$

**4** TI-Nspire CAS





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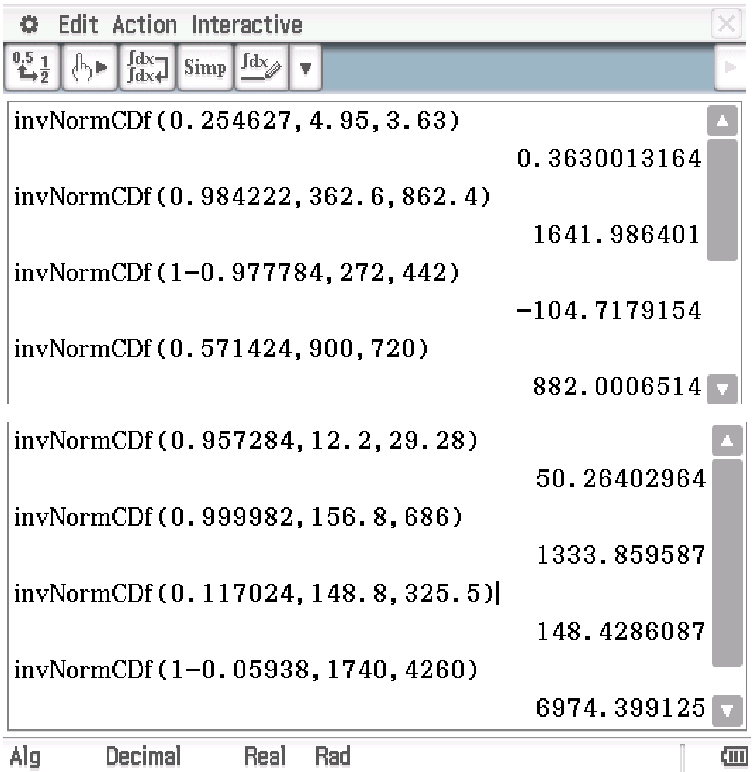
## ClassPad




**a** $\mu = 3.63$, $\sigma = 4.95$

$P(-\infty < X < a) = 0.254\ 627$

$a = 0.363$

**b** $\mu = 862.4$, $\sigma = 362.6$

$P(a < X < 2262.036) = 0.015\ 721$

$0.015\ 721 = P(X < 2262.036) - P(X < a)$

$0.015\ 721 = 0.999\ 9433 - P(X < a)$

$P(X < a) = 0.984\ 222$

$a = 1641.99$

**c** $\mu = 442$, $\sigma = 272$

$P(a < X < \infty) = 0.977\ 784$

$a = -104.72$

**d** $\mu = 720$, $\sigma = 900$

$P(-\infty < X < a) = 0.571\ 424$

$a = 882$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**e** $\mu = 29.28$, $\sigma = 12.2$

$P(13.664 < X < a) = 0.857\ 011$

$0.857\ 011 = P(X < a) - P(X < 13.664)$

$0.857\ 011 + 0.100\ 273 = P(X < a)$

$P(X < a) = 0.957\ 284$

$a = 50.26$

**f** $\mu = 686$, $\sigma = 156.8$

$P(618.576 < X < a) = 0.666\ 384$

$0.666\ 384 = P(X < a) - P(X < 618.576)$

$0.666\ 384 + 0.333\ 597 = P(X < a)$

$P(X < a) = 0.999\ 9818$

$a = 1333.5$

**g** $\mu = 325.5$, $\sigma = 148.8$

$P(a < X < 859.692) = 0.882\ 811$

$0.882\ 811 = P(X < 859.692) - P(X < a)$

$P(X < a) = 0.999\ 835 - 0.882\ 811$

$P(X < a) = 0.117\ 024$

$a = 148.43$

**h** $\mu = 4260$, $\sigma = 1740$

$P(a < X < \infty) = 0.059\ 38$

$a = 6974.4$

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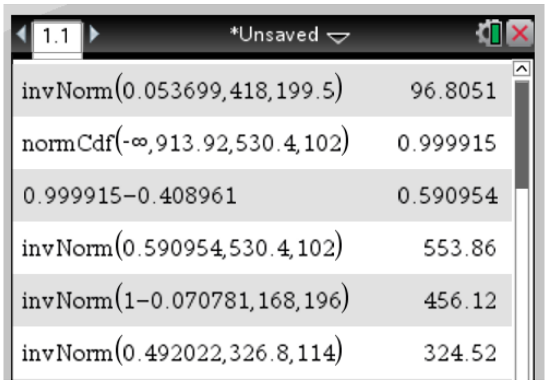
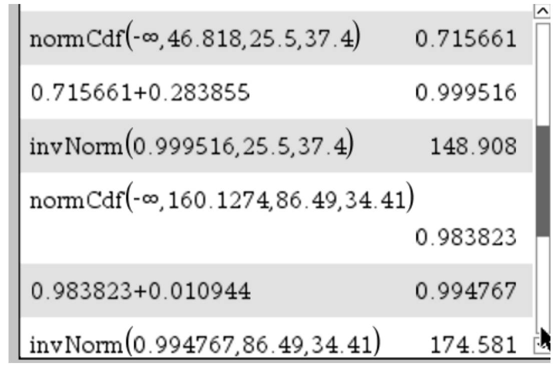
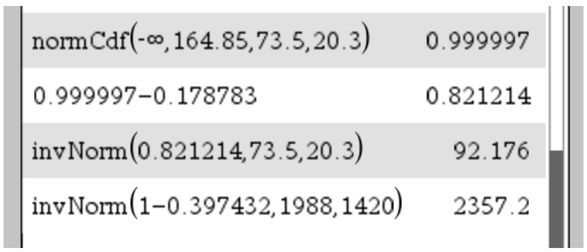
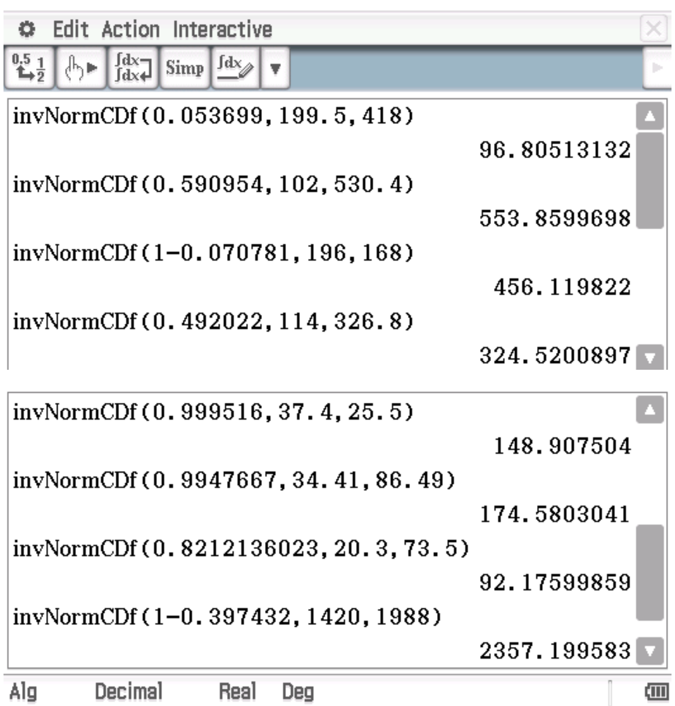
**5** **TI-Nspire CAS**




**ClassPad**

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**a** $\mu = 418, \sigma = 199.5$

$P(-\infty < X < a) = 0.053\ 699$

$a = 96.81$

**b** $\mu = 530.4, \sigma = 102$

$P(a < X < 913.92) = 0.408\ 961$

$0.408\ 961 = P(X < 913.92) - P(X < a)$

$P(X < a) = P(X < 913.92) - 0.408\ 961$

$P(X < a) = 0.999\ 915 - 0.408\ 961$

$P(X < a) = 0.590\ 954$

$a = 553.86$

**c** $\mu = 168, \sigma = 196$

$P(a < X < \infty) = 0.070\ 781$

$a = 456.12$

**d** $\mu = 326.8, \sigma = 114$

$P(-\infty < X < a) = 0.492\ 022$

$a = 324.52$

**e** $\mu = 25.5, \sigma = 37.4$

$P(46.818 < X < a) = 0.283\ 855$

$0.283\ 855 = P(X < a) - P(X < 46.818)$

$P(X < 46.818) + 0.283\ 855 = P(X < a)$

$P(X < a) = 0.715\ 661 + 0.283\ 855$

$P(X < a) = 0.999\ 516$

$a = 148.92$

**f** $\mu = 86.49, \sigma = 34.41$

$P(160.1274 < X < a) = 0.010\ 944$

$0.010\ 944 = P(X < a) - P(X < 160.1274)$

$P(X < 160.1274) + 0.010\ 944 = P(X < a)$

$P(X < a) = 0.983\ 8226 + 0.010\ 944$

$P(X < a) = 0.994\ 7667$

$a = 174.58$

**g** $\mu = 73.5$, $\sigma = 20.3$

$P(a < X < 164.85) = 0.178\ 783$

$0.178\ 783 = P(X < 164.85) - P(X < a)$

$P(X < a) = 0.999\ 9966 - 0.178\ 783$

$P(X < a) = 0.821\ 213\ 6023$

$a = 92.176$

**h** $\mu = 1988$, $\sigma = 1420$

$P(a < X < \infty) = 0.397\ 432$

$a = 2357.2$

## Reasoning and communication

**6**

| English: | Maths Methods: |
|---|---|
| 18 out of 25 | 15 out of 20 |
| $\mu = 15$ | $\mu = 13$ |
| $\sigma = 8$ | $\sigma = 5$ |

$$z = \frac{X - \mu}{\sigma}$$

| English | Maths Methods |
|---|---|
| $z = \frac{18-15}{8}$ | $z = \frac{15-13}{5}$ |
| $z = 0.375$ | $z = 0.4$ |

Callum did relatively better in Maths Methods.

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**7** Height: IQ:

175 cm 110

$\mu = 171$ cm $\mu = 100$

$\sigma = 12$ $\sigma = 15$

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{175 - 171}{12} \qquad z = \frac{110 - 100}{15}$$

$$z = 0.33 \qquad z = 0.67$$

Deirdre's IQ is further away from the average than her height.

**8** Player 1: Player 2:

$\mu = 25$ points $\mu = 29$ points

$\sigma = 9$ $\sigma = 5$

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{37 - 25}{9} \qquad z = \frac{37 - 29}{5}$$

$$z = 1.33 \qquad z = 1.6$$

The first player is more likely to score more than 37 points in a particular game, as the *z*-score is closer to the mean.

**9** $\mu = 11.3$ $\sigma = 5.4$

$$z = \frac{X - \mu}{\sigma}$$

$X = 2$ $X = 17$

$$z = \frac{2 - 11.3}{5.4} \qquad z = \frac{17 - 11.3}{5.4}$$

$$z = -1.74 \qquad z = 1.0\overline{5}$$

It is more unusual for ten-year-old girls to be able to do only 2 push-ups.

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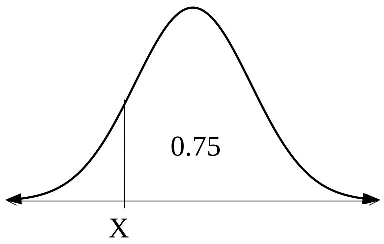
**10** $\mu = 45$, $\sigma = 13.7$




$P(X > p) = 0.75$

$p = 35.76$

The pass mark should be 35 to make sure 75% pass.

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## Exercise 8.09 Using the normal distribution

### Reasoning and communication

**1** $\mu = 60$ days, $\sigma = 19$ days

**a** $P(X > 90) = 0.057$

5.7%

**b** $P(X < 30) = 0.057$

5.7%

**c** $P(X < 10) = 0.004\ 25$

0.42%

**2** $\mu = 268.5$ mm, $\sigma = 84.5$ mm

**a** $P(X > 220 \text{ mm}) = 0.717$

**b** $P(X < 190 \text{ mm}) = 0.176$

**3** $\mu = \$36/\text{h}$, $\sigma = \$7$

**a** $P(X > \$43) = 0.1587$

$E(X) = n \times P(X)$

$E(X) = 20 \times 0.1587$

$E(X) = 3.17$, i.e. 3

**b** $P(X < \$38) = 0.6125$

$E(X) = n \times P(X)$

$E(X) = 50 \times 0.6125$

$E(X) = 30.62$, i.e. 31


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**4** $\mu$ = 40 mins, $\sigma$ = 3 mins

Leaving at 8:15 a.m., her mean time of arrival would be 8:55 a.m.

Count her starting time, 9 a.m., as 0.

Then $\mu$ = –5 mins, $\sigma$ = 3 mins

Assume 9:00 a.m. means 8:59:30 til 9:00:30, etc. i.e. $-0.5 < t < 0.5$

**a** $P(-0.5 < t < 0.5) = 0.0334$

**b** $P(-1.5 < t < -0.5) = 0.0549$

**c** $P(0.5 < t < 1.5) = 0.0182$

**d** $P(-2.5 < t < -1.5) = 0.0807$

**e** $P(1.5 < t < 2.5) = 0.0089$

**f** $P(\text{early}) = P(t < 0) = 0.9522$

**g** $P(\text{late}) = P(t > 0) = 0.0478$

**5** $\mu$ = 4500 hours, $\sigma$ = 400 hours

Assume the measurements are rounded to the nearest 100.

**a** $P(X = 4500) = P(4450 < X < 4550) = 0.0995$

**b** $P(X = 4000) = P(3950 < X < 4050) = 0.0457$

**c** $P(X = 4800) = P(4750 < X < 4850) = 0.0752$

**d** $P(X = 5000) = P(4950 < X < 5050) = 0.0457$

**e** $P(X = 4100) = P(4050 < X < 4150) = 0.0605$

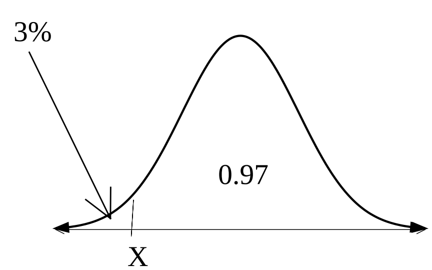
**6** $\mu = 15.5°$, $\sigma = 2.6°$

Assume measurements are made correct to one decimal place.

$P(15°) \approx P(15.45° < T < 15.55°) = 0.015$

$P(15°) \approx P(14.5° < T < 15.5°) = 0.1497$

**7** $\mu$ = 26 months, $\sigma$ = 2.5 months




$P(X > t) = 0.03$

$t$ = 21 months

**8** $\mu = 10$, $\sigma = 2.7$

**a** We want $S$ so that $P(X > S) = 0.05$, i.e. $t_{0.95}$

Using the inverse normal, $S = 14.44\ldots$

For a whole number, they need to get 15 or more for it to be a less than 5% chance.

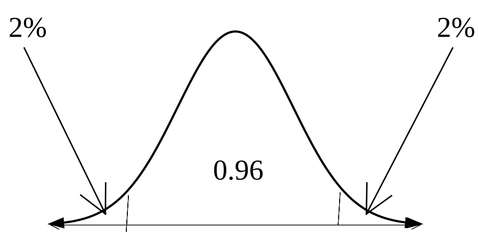
**b** Considering a random situation with each person having a probability of exceeding the threshold of $p = 0.05$, this is a binomial probability situation, so the probability of getting 7 or more exceeding the threshold is $1 - P(X < 8)$.

Using the cumulative binomial distribution, $1 - P(X < 8) = 0.128 = 12.8\%$

Since there is a 12.8% chance of getting the result by chance, she has not found evidence within a probability of 5%.

**9** Supposed to be 40 mm

$\mu = 40$ mm, $\sigma = 0.6$ mm




$P(l < X) = 0.98$

Too long: $l = 41.2322$

Too short: $l = 38.7677$

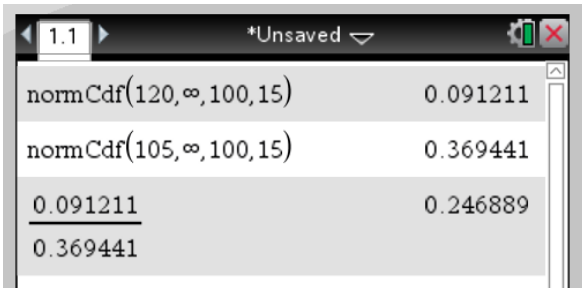
The range is 38.77 mm to 41.23 mm.

**10**

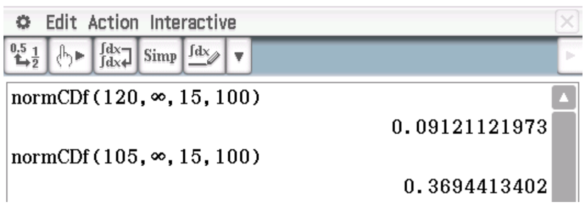
| Weldon: | Betterdon: |
|---|---|
| $\mu = 1048$ mm | $\mu = 839$ mm |
| $\sigma = 255$ mm | $\sigma = 122$ mm |
| $P(X < 500 \text{ mm}) = ?$ | |
| $P(X < 500 \text{ mm}) = 0.0158$ | $P(X < 500 \text{ mm}) = 0.0027$ |

It is more likely that the annual rainfall will fall below 500 mm in Weldon.

**11** TI-Nspire CAS




ClassPad




**a** Using normcdf(120, ∞, 100, 15), $P(\text{IQ} > 120) \approx 0.0912$

**b** Using normcdf(105, ∞, 100, 15), $P(\text{IQ} > 105) \approx 0.3694$

Also $P(\text{IQ} > 120 \text{ and IQ} > 105) = 0.0912$

So $P(\text{IQ} > 120 \mid \text{IQ} > 105) = \dfrac{0.0912}{0.3694} \approx 0.2469$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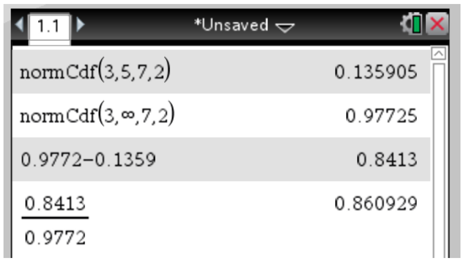
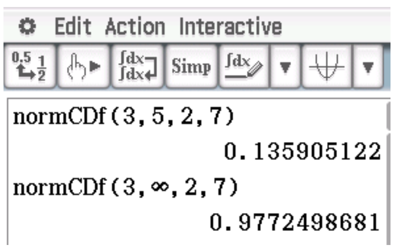
**12** TI-Nspire CAS




ClassPad




**a** Using normcdf(3, 5, 7, 2), $P(\text{chat}) \approx 0.1359$

**b** Using normcdf(3, ∞, 7, 2), $P(\text{packed}) \approx 0.9772$

$P(\text{normal}) \approx 0.9772 - 0.1359 = 0.8413$

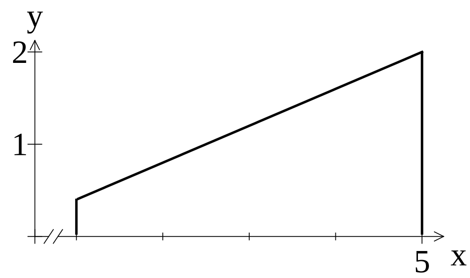
$P(\text{normal} \mid \text{packed}) = 0.8413 \div 0.9772 \approx 0.8609$

# Chapter 8 Review

## Multiple choice

**1** B

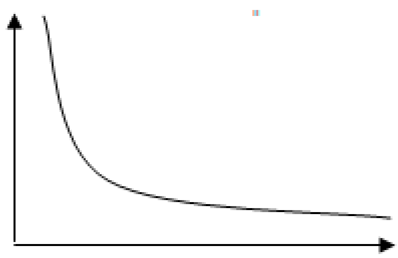
I $f(x) = 0.4x$




Area = $\int_1^5 0.4x\,dx = 0.2\left[x^2\right]_1^5 = 0.2(25-1) = 4.8$

Not a probability density function as the area on the defined domain is not equal to one.

II $f(x) = \dfrac{1}{x\ln(5)}\left(=\dfrac{k}{x}\right)$



Area = $\int_1^5 \frac{1}{x\ln(5)}dx = \frac{1}{\ln(5)}\int_1^5 \frac{1}{x}dx = \frac{1}{\ln(5)}\left[\ln(x)\right]_1^5 = \frac{1}{\ln(5)}\left(\ln(5)-\ln(1)\right) = 1$

Always positive and area is 1 on the defined domain so it is a probability density function.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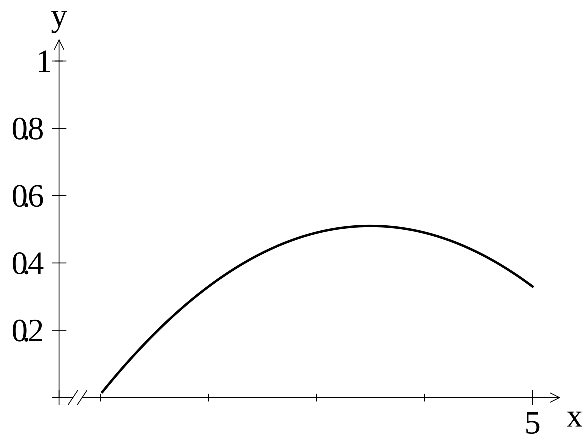
III $f(x) = 0.56x - 0.08x^2 - 0.47$



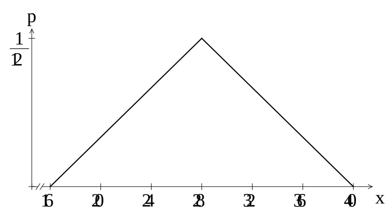

Area = $\int_1^5 0.56x - 0.08x^2 - 0.47dx = 1.5\overline{3}$

Not a probability density function as the area on the defined domain is not equal to one.

**2** A as $\frac{d}{dx}\left(0.5x^3\right) = 0.15x^2$

**3** E [4, 44], as $E(X)$ is in the middle due to symmetry and $\frac{4+44}{2} = 24$

**4** E [16, 40]




$$p(x) = \begin{cases} \frac{x}{144} - \frac{1}{9} & \text{for } 16 \le x \le 28 \\ -\frac{x}{144} + \frac{5}{18} & \text{for } 28 \le x \le 40 \end{cases}$$

$E(x) = 28$ from symmetry

$$Var(X) = \int_a^b p(x)(x-\mu)^2 dx$$

$$Var(X) = \int_{16}^{28} \left(\frac{x}{144} - \frac{1}{9}\right)(x-28)^2 dx + \int_{28}^{40} \left(-\frac{x}{144} + \frac{5}{18}\right)(x-28)^2 dx$$

$$= 24$$

**5** D $P(0 \le Z \le 2.14) = 0.4838$

**6** B $\mu = 24$, $\sigma = 5$, $X = 28$, $z = ?$

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{28 - 24}{5}$$

$$z = 0.8$$

**7** B $\mu = 21$, $\sigma = 5.3$

$P(19 < X < 22) = 0.2219$

**8** D $\mu = 50$, $\sigma = 12$,

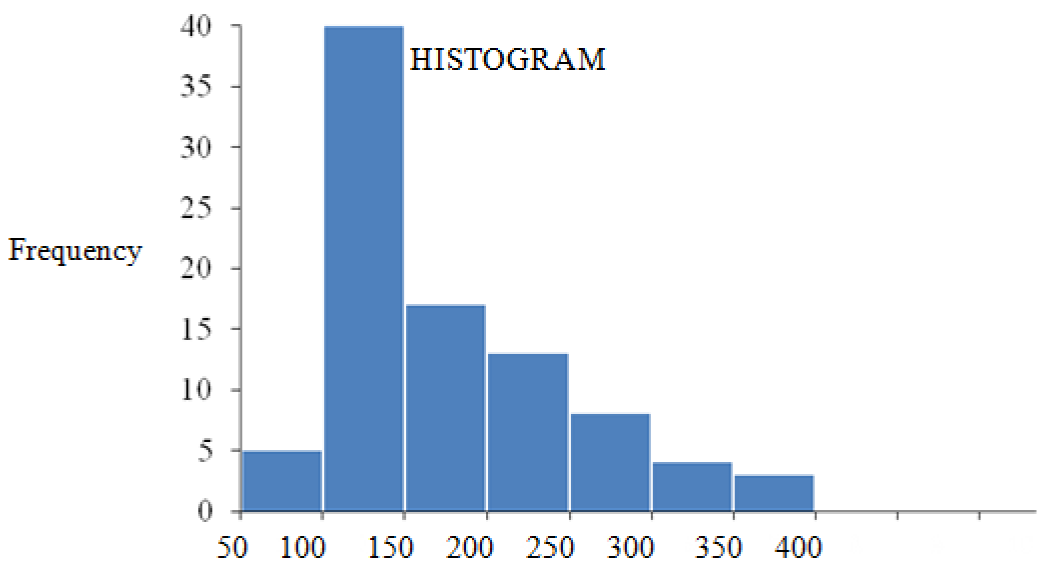
$P(X < g) = 0.3$

$g = 43.71$

## Short answer

**9** Using 0–50, 51–100, etc,

| **Data** | **Frequency** |
|---|---|
| $50.5 < x < 100.5$ | 5 |
| $100.5 < x < 150.5$ | 40 |
| $150.5 < x < 200.5$ | 17 |
| $200.5 < x < 250.5$ | 13 |
| $250.5 < x < 300.5$ | 8 |
| $300.5 < x < 350.5$ | 4 |
| $350.5 < x < 400.5$ | 3 |
| **Total** | **90** |




The edges are actually at 50.5, 100.5, etc.

$$P(179.5 \le X \le 220.5) = \frac{\frac{21}{50}\times 17 + \frac{2}{5}\times 13}{90} = 0.1371...$$

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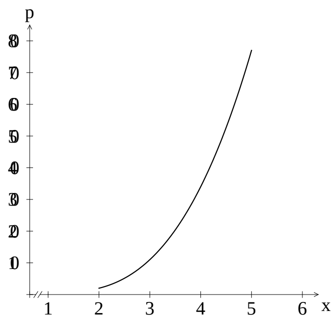
**10** $f(x) = x^3 - 2x^2 + 2$ on [2, 5].



$$\int_2^5 x^3 - 2x^2 + 2\,dx = 80.25 = \frac{321}{4}$$

$p(x) = \dfrac{4}{321}(x^3 - 2x^2 + 2)$

**11** $p(x) = \dfrac{1}{6}$

$P(2.25 \le x < 2.35) = \dfrac{1}{60}$

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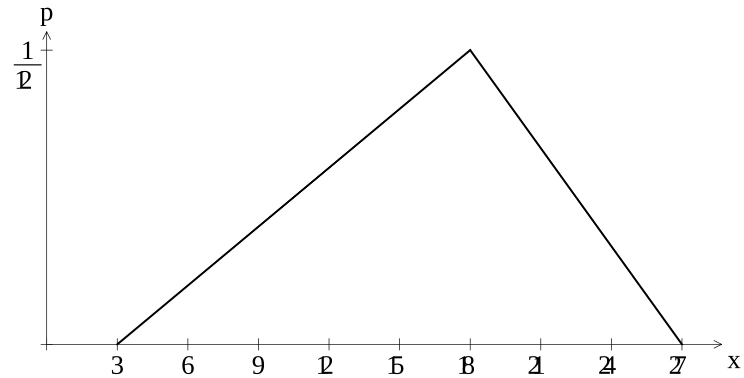
**12** [3, 27], a maximum value at 18.

Width of base is 24.

Using [3, 27], $h = \frac{1}{12}$




$$p(x) = \begin{cases} \frac{x}{180} - \frac{1}{60} & \text{for } 3 \le x \le 18 \\ -\frac{x}{108} + \frac{1}{4} & \text{for } 18 \le x \le 27 \end{cases}$$

$$E(x) = \int_3^{18} x \times \left(\frac{x}{180} - \frac{1}{60}\right) dx + \int_{18}^{27} x \times \left(-\frac{x}{108} + \frac{1}{4}\right) dx$$

$$= 8.125 + 7.875$$

$$= 16$$

**13** $E(X) = 27.8$

$SD(X) = 5.6$

$Y = 2X + 3$

$E(Y) = 2E(X) + 3 = 58.6$

$SD(Y) = 2SD(X) = 11.2$

**14** $\mu = 76$ and $\sigma = 5.2$

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = 0.0767 e^{-0.0185(x-76)^2}$$

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**15** English:

$\mu = 18.8$

$\sigma = 5.4$

$$z = \frac{X - \mu}{\sigma}$$

$$z = \frac{27 - 18.8}{5.4}$$

$z = 1.52$

Maths Methods:

$\mu = 22.3$

$\sigma = 3.6$

$$z = \frac{27 - 22.3}{3.6}$$

$z = 1.31$

Danielle did relatively better on the English test.

**16** **a** $P(M > -0.7) = 0.758$

**b** $P(0.2 \le M \le 2.4) = P(M \le 2.4) - P(M \le 0.2)$

$= 0.4125$

**17** $\mu = 124\ 000$, $\sigma = 38\ 000$

$P(x < X) = 0.25$

$x = 98\ 360 \approx 98\ 000$ km

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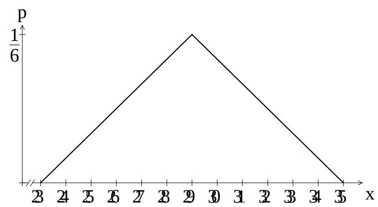
## Application

**18** $p(x) = \frac{1}{15}$

$P(5 < X < 8) = \frac{5}{15} = \frac{1}{5} = 0.2$

**19**




$$p(x) = \begin{cases} \frac{x}{36} - \frac{23}{36} & \text{for } 23 \le x \le 29 \\ -\frac{x}{36} + \frac{35}{36} & \text{for } 29 \le x \le 35 \end{cases}$$

$P(x = 30) = \int_{29.5}^{30.5} \left(-\frac{x}{36} + \frac{35}{36}\right) dx = 0.13\overline{8}$

**20** $\mu = 25$ mins

$P(X < 20) = 0.3$

$z = -0.5244$

$z = \frac{X - \mu}{\sigma}$

$-0.5244 = \frac{20 - 25}{\sigma}$

$\sigma = 9.5347$

$P(X > 28) = 0.3765$